H2 Mathematics 2016 Paper 2 Revision

1 The curve C has equation

$$f(x)=\frac{ax^2+bx+1}{x+c}, \text{ where } a, b \text{ and } c \text{ are real constants.}$$

Given that the line $y=2x-1$ is an asymptote of C, find the value of a and show that $b=2c-1$. [3]

(i) For $c=1$, using algebraic method, prove that the curve C cannot lie between 2 values, which are to be determined. [3]

(ii) Sketch the graph of $f(x)=\frac{2x^2+x+1}{x+1}$, showing clearly its asymptotes, the coordinates of the axial intercepts, and turning point(s) (if any). [3]

Hence, state the range of x for which $f(x)$ is concaving downwards. [1]

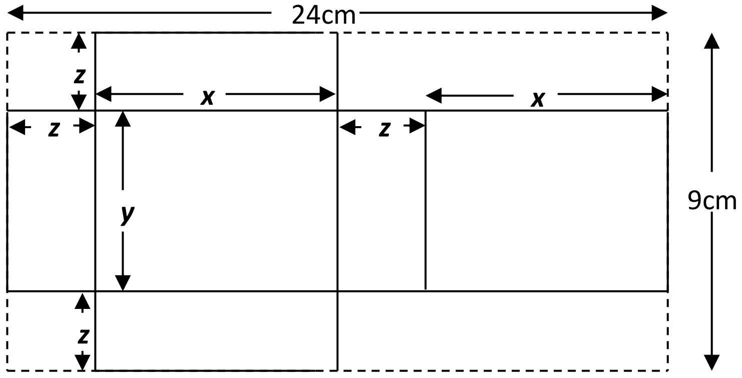
(iii) Given that the line $y=kx+k-3$, where k is a real constant, passes through the intersection of the asymptotes of C, deduce the range of k where

$$2x^2+x+1=(kx+k-3)(x+1)$$

has 2 real solutions. [1]

(iv) Without differentiation, sketch the graph of f '(x) and determine the interval whereby the graph is concave upwards. [3]

2 The figure below shows a 24cm by 9cm cardboard sheet. It can be folded into a box with length x, width y and height z.

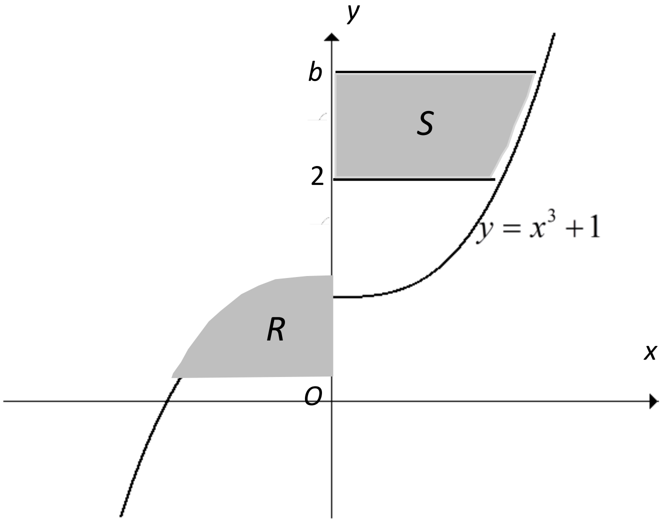


http://alevelmathtuition.sg
94385929 / 81502027

(i) Show that the volume of the box, V cm^3, can be expressed as $V = -2x^3 + 39x^2 - 180x$. [3]

(ii) Without the use of graphing calculator, find the largest volume of the box. [4]

3



(i) The diagram above shows the curve with equation $y = x^3 + 1$. Given that the two shaded areas R and S have the same value, find the value of b. [4]

(ii) Find the volume of the solid generated when S is rotated completely about the x-axis. [4]

4 **(a)** Find $\int \frac{e^t}{(1+3e^t)^2}\, dt$. [2]

(b) Write down $\frac{d}{dx}\left(\tan(x^2)\right)$. Hence find $\int x^3 \sec^2\left(x^2\right) dx$. [4]

(c) Evaluate $\int_0^4 x^2 |x-3| \, dx$ without the use of the graphic calculator. [3]

5 The curve C has parametric equations

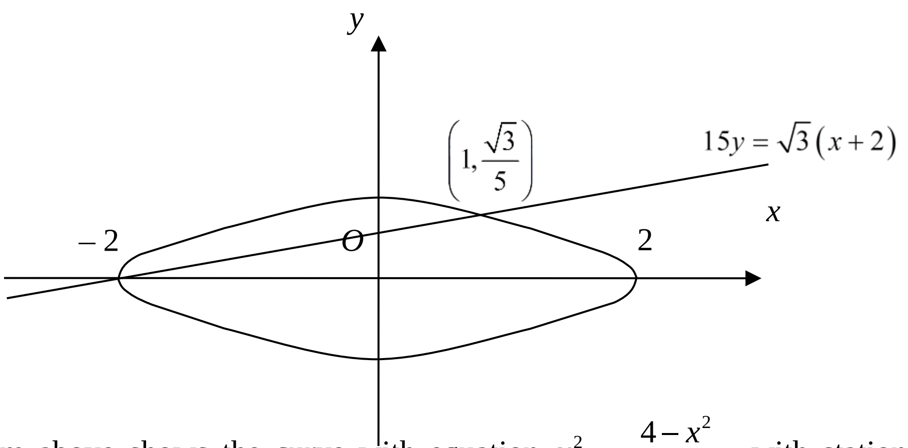
$$x = e^{\theta}\cos\theta, \qquad y = \sin\theta + \cos\theta, \qquad 0 \le \theta \le \frac{\pi}{4}.$$

(i) Show that the equation of the tangent at $\left(\frac{\sqrt{3}}{2}e^{\frac{\pi}{6}}, \frac{\sqrt{3}+1}{2}\right)$ is given by $y = e^{-\frac{\pi}{6}}x + \frac{1}{2}$. [4]

(ii) Show that the area bounded by the curve C and the x-axis can be expressed as $\int_0^{\frac{\pi}{4}} e^{\theta}\cos 2\theta \, d\theta$. Hence, evaluate the area, leaving your answer correct to 2 decimal places. [4]

6 **(i)** Use the substitution $x = 2\tan\theta$ to show $\int \frac{4-x^2}{\left(4+x^2\right)^2}\,dx = \frac{x}{4+x^2} + C$, where C is an arbitrary constant. [5]

(ii)



The diagram above shows the curve with equation $y^2 = \frac{4-x^2}{\left(4+x^2\right)^2}$ with stationary points at $x = 0$. The line $15y = \sqrt{3}(x+2)$ intersects the curve at $(-2, 0)$ and $\left(1, \frac{\sqrt{3}}{5}\right)$.

(iii) The region bounded by the curve $y = \sqrt{\frac{4-x^2}{\left(4+x^2\right)^2}}$ and the line $15y = \sqrt{3}(x+2)$ is rotated through 4 right angles about the x-axis to form a solid of revolution of volume V. Find the exact value of V, giving your answer in the form $b\pi$. [4]

7 On a single Argand diagram, sketch the following loci.

(i) $|z-5| = |3+\sqrt{7}\mathrm{i}|$, [1]

(ii) $|z-6-\sqrt{3}\mathrm{i}| = |z-4+\sqrt{3}\mathrm{i}|$. [1]

Two complex numbers that satisfy the above equations are represented by p and q, where $\mathrm{Re}(p) < \mathrm{Re}(q)$. By using the cartesian equations of the loci, find p and q. Hence, determine the value of $\arg(p-q)$. [5]

8 (i) Solve the equation

$$z^5 - 32 = 0,$$

expressing the answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. [2]

(ii) Explain why the equation $\left(\frac{2w+1}{w}\right)^5 - 32 = 0$ has four roots. [1]

The roots of the equation are denoted by w_1, w_2, w_3 and w_4. By finding $\frac{1}{w}$, show that $\sum_{i=1}^{4} \frac{1}{w_i}$ is a real number. [4]

9 On a single Argand diagram, sketch the loci given by

(i) $|z-1-\mathrm{i}|^2 \ge 2$,

(ii) $\arg\left(\frac{z+1}{\sqrt{3}+\mathrm{i}}\right) \ge \frac{\pi}{12}$,

(iii) $|z| > |z-1|$. [7]

Hence, or otherwise, find the range of values of $|z-\mathrm{i}|$ and $\arg(z-\mathrm{i})$. [3]