Vectors

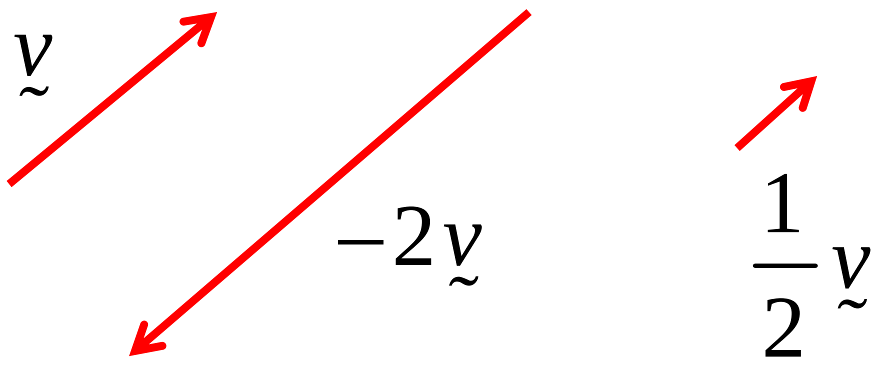
Parallel vectors and collinear points

Parallel vectors

- Two vectors $\underset{\sim}{v}$ and $\underset{\sim}{w}$ are **parallel** if

$$\underset{\sim}{v} = k\underset{\sim}{w}$$

for non – zero constant k

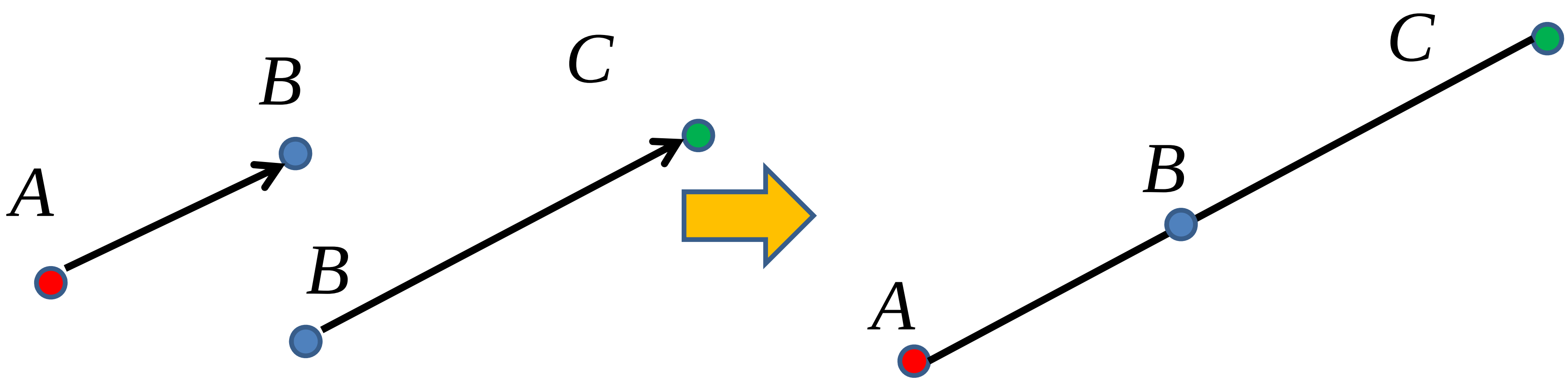


Collinear points

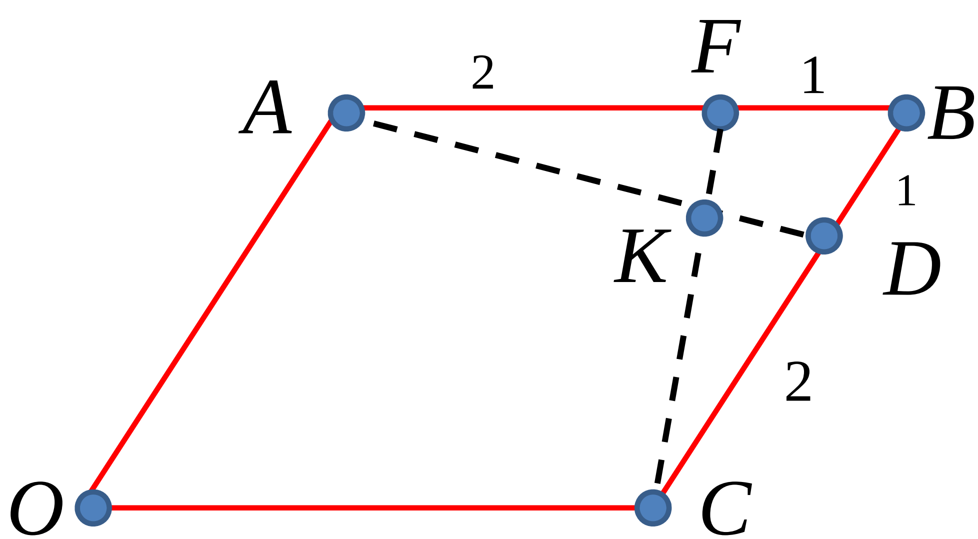
- 3 points, A, B and C are <u>**collinear**</u> if

$$\overrightarrow{AB} = k\,\overrightarrow{BC}$$

for non – zero constant k



OABC is a parallelogram and *D* and *F* are points on *CB* and *AB* respectively, such that *CD* : *DB* = 2 : 1 and *AF* : *FB* = 2 : 1. Given that *AD* and *CF* intersect at *K*, by expressing vectors in terms of $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$, show that $AK = \frac{3}{4}AD$.

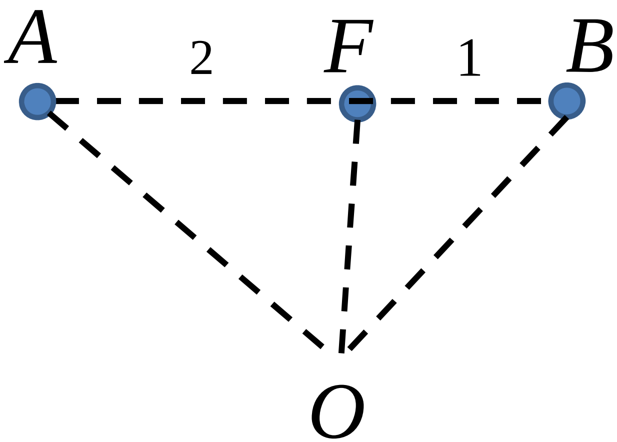


Step 1: Find $\overrightarrow{OF}$ and $\overrightarrow{OD}$.

Step 2: Use the fact that *A*, *K*, *D* and *C*, *K*, *F* are collinear to find $\overrightarrow{OK}$.

Step 3: Calculate $\overrightarrow{AK}$ and $\overrightarrow{AD}$.

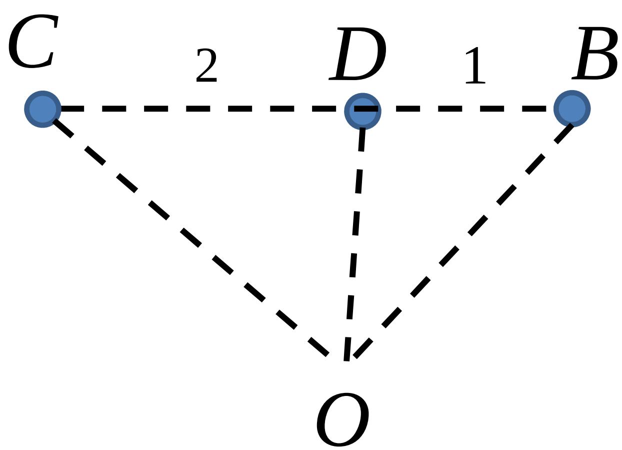
Step 1: Find $\overrightarrow{OF}$ and $\overrightarrow{OD}$.



Using ratio theorem

$$\overrightarrow{OF} = \frac{\overrightarrow{OA} + 2\overrightarrow{OB}}{3} = \frac{\underset{\sim}{a} + 2(\underset{\sim}{c} + \underset{\sim}{a})}{3}$$

$$= \underset{\sim}{a} + \frac{2}{3}\underset{\sim}{c}$$



$$\overrightarrow{OD} = \frac{\overrightarrow{OC} + 2\overrightarrow{OB}}{3} = \frac{\underset{\sim}{c} + 2(\underset{\sim}{c} + \underset{\sim}{a})}{3}$$

$$= \underset{\sim}{c} + \frac{2}{3}\underset{\sim}{a}$$

Step 2: Use the fact that A, K, D and C, K, F are collinear to find $\overrightarrow{OK}$.

$$\overrightarrow{AK} = \lambda \overrightarrow{AD}$$

$$\overrightarrow{OK} - \underset{\sim}{a} = \lambda\left(\underset{\sim}{c} + \frac{2}{3}\underset{\sim}{a} - \underset{\sim}{a}\right)$$

$$\overrightarrow{OK} = \lambda \underset{\sim}{c} + \left(1 - \frac{\lambda}{3}\right)\underset{\sim}{a} \quad (1)$$

$$\overrightarrow{CK} = \mu \overrightarrow{CF}$$

$$\overrightarrow{OK} - \underset{\sim}{c} = \mu\left(\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{c} - \underset{\sim}{c}\right)$$

$$\overrightarrow{OK} = \mu \underset{\sim}{a} + \left(1 - \frac{\mu}{3}\underset{\sim}{c}\right) \quad (2)$$

$$(1) = (2)$$

$$\lambda \underset{\sim}{c} + \left(1 - \frac{\lambda}{3}\right)\underset{\sim}{a} = \mu \underset{\sim}{a} + \left(1 - \frac{\mu}{3}\underset{\sim}{c}\right)$$

Comparing coeff

$$\lambda = 1 - \frac{\mu}{3}, \quad \mu = 1 - \frac{\lambda}{3}$$

$$\lambda = \mu = \frac{3}{4}$$

$$\therefore \overrightarrow{OK} = \frac{3}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{c}$$

Step 3: Calculate $\overrightarrow{AK}$ and $\overrightarrow{AD}$.

$$\overrightarrow{AK} = \overrightarrow{OK} - \overrightarrow{OA}$$

$$= \frac{3}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{c} - \underset{\sim}{a}$$

$$= \frac{3}{4}\underset{\sim}{c} - \frac{1}{4}\underset{\sim}{a}$$

$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$$

$$= \frac{2}{3}\underset{\sim}{a} + \underset{\sim}{c} - \underset{\sim}{a}$$

$$= \underset{\sim}{c} - \frac{1}{3}\underset{\sim}{a}$$

$$\Rightarrow \overrightarrow{AK} = \frac{3}{4}\overrightarrow{AD}$$

$$\therefore AK = \frac{3}{4} AD$$