HWA CHONG INSTITUTION
2016 JC2 PRELIMINARY EXAMINATION

MATHEMATICS
Higher 2

9740/02

Paper 2

Tuesday **20 September 2016** **3 hours**

Additional materials: Answer paper
List of Formula (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT class on all the work you hand in, including the Cover Page.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Do not write anything on the List of Formula (MF15).

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, place the completed cover page on top of your answer scripts and fasten all your work securely together with the string provided.

This question paper consists of 6 printed pages.

Section A: Pure Mathematics [40 marks]

1. On the first day of last month, the temperature of a machine in a manufacturing plant was found to be $90\,°\text{C}$ and was deemed as too hot by the supervisor. On each day waste heat was produced by the machine as a by-product and caused the temperature of the machine to increase by $2\,°\text{C}$. In an attempt to make it cooler, the supervisor decided to adjust the thermostat and decreased the temperature by 3% at the end of each day.

(i) Show that the temperature of the machine was 87.8 °C at the end of the 3rd day. [2]

(ii) At the end of which day would the temperature of the machine first dropped below $70°\text{C}$? [3]

(iii) Will the temperature continue to drop indefinitely? Justify your answer. If not, what is the long term temperature of the machine? [3]

2. **(a)** Given that $z_1 = -\dfrac{\text{i}}{2}$ is a root of the equation $2z^3 + (\text{i} - 8)z^2 + az + 13\text{i} = 0$, find the complex number a and solve the equation, giving your answer in Cartesian form $x + \text{i}y$. [4]

Hence, find in Cartesian form the roots of the equation

$$2w^3 + (1 + 8\text{i})w^2 - aw - 13 = 0.$$ [2]

(b) Solve the equation $z^6 + 729 = 0$, expressing your answers in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

Given that z_1 is a root of the above equation and $0 < \arg z_1 < \dfrac{\pi}{2}$.

If $\dfrac{{z_1}^n}{z_1^*}$ is a positive real number, find the smallest positive integer n. [2]

3. **(i)** Use the substitution $x = 3\sin\theta + 1,\ 0 < \theta < \dfrac{\pi}{2}$ to find $\displaystyle\int \frac{x}{\sqrt{9 - (x-1)^2}}\,\text{d}x$. [4]

(ii) A curve has parametric equations

$$x = \frac{1}{\sqrt{9 - (t-1)^2}}, \quad y = t^2, \quad 0 < t < 4.$$

(a) Sketch the curve, indicating the end point and the equation of the asymptote. [3]

(b) Using the result in part **(i)**, find the exact area of the region bounded by the curve, the lines $y = 1,\ y = \dfrac{25}{4}$ and $x = \dfrac{8}{7}$. [4]

4. In harvesting of renewable natural resources, it is desirable that policies are formulated to allow maximal harvest of the natural resources, and yet not deplete the resources below a sustainable level. A simple harvesting model devised for the rate of change of the population of wild salmon in a particular region in the Pacific Ocean is given by

$$\frac{\mathrm{d}P}{\mathrm{d}t} = P(4-P) - h,$$

where P is the population of wild salmon in millions at time t years and h is the constant harvest rate in millions.

(i) Sketch a graph of $\frac{\mathrm{d}P}{\mathrm{d}t}$ against P, expressing the turning point in terms of h. [2]

(ii) The *Maximum Sustainable Yield (MSY)* is the largest harvest rate h that allows for a sustainable harvest of wild salmon without long-term depletion. State the *MSY* for the wild salmon. [1]

(iii) It is given that the population of wild salmon in that region was 3.2 million in 2015 and the constant harvest rate is 3 million. Find an expression for P at any time t. [5]

Hence find the population of the wild salmon in that region in 2016. [2]

(iv) State one assumption you made in your calculation. [1]

Section B: Statistics [60 marks]

5. In the last election, there were speculations from unofficial sources before the counting of votes is completed. For the current election, to prevent unnecessary speculations, the election office of Sunny Island will be conducting a sample count in each electoral division after voting is done. Each electoral division has a different number of registered voters and a sample of 400 votes will to be sampled from each electoral division.

(i) Identify and describe an appropriate method to obtain the sample. [3]

(ii) State an advantage and a disadvantage of the sampling method used in part **(i)**. [2]

6. The random variable X has a binomial distribution $\mathrm{B}(n, p)$, where $0 < p < 1$, and n is an integer. Show that $\dfrac{\mathrm{P}(X = r)}{\mathrm{P}(X = r-1)} = \left(\dfrac{n-r+1}{r}\right)\left(\dfrac{p}{1-p}\right)$. [3]

Hence find a condition relating n and p such that X has two values for its mode, and determine these two values, giving your answer in terms of n and p. [3]

7. A group of ten people consists of four single women, two single men and 2 couples. The ten people are arranged randomly in a circle.

(i) Find the probability that the four single women are all separated. [2]

(ii) Find the probability that either the four single women are next to one another or the two single men are next to each other but not both. [3]

One of the ten people left the group and the remaining nine decided to sit at a round table with ten identical chairs equally spaced around the table. The chairs are decorated such that every alternate chair is tied with an identical chair sash. Given that the nine people have no preference to which seat to take, find the number of possible seating arrangements. [2]

8. In an examination, the score, X, for paper 1 of a student is found to follow a normal distribution with mean 62 and standard deviation σ, and the score, Y, for paper 2 of a student is found to follow a normal distribution with mean 71 and standard deviation 8. The final score of a student for the examination is the average score of the 2 papers and it is assumed that X and Y are independent random variables.

(i) Find the probability that for two randomly selected students A and B taking the examination for paper 2, A has at most 2 marks less than the marks of B. [2]

(ii) Given that 15% of the students have at least a final score of 75, find σ. [4]

(iii) Using the value of σ found in part **(ii)**, find the probability that a randomly selected student performs better in her Paper 1 than in her Paper 2.

(iv) Comment on the validity of the answer obtained in part **(ii)** and **(iii).** [3]

9. To reduce the number of speeding incidents on the road, traffic police in Country S set up traffic cameras at 3 busy traffic Junctions A, B and C to monitor the speeds of vehicles passing through these junctions. The average number of speeding vehicles caught by the camera at Junctions A, B and C are 2 in every 3 hours, 5 in every 4 hours and λ in every hour respectively. It is assumed that the number of speeding vehicles caught by the cameras at the three junctions followed Poisson distributions.

(a) Find the probability that there are at least 2 speeding vehicles caught at Junctions A and B in an hour. [2]

(b) Given that there are 2 speeding vehicles caught at the three junctions in an hour, find the probability that at least one speeding vehicle caught is at Junction C. Leave your answer in terms of λ. [3]

(c) Given that the traffic cameras are in operation 24 hours in a day, using a suitable approximation, find the probability that there will be more speeding vehicles caught at Junction A than at Junction B in a day. State an assumption for the calculation to be valid. [5]

10. A nutritionist claims that the mean number of calories in an energy bar is 350 cal. The nutritionist collected and measured the number of calories of a random sample of 15 energy bars. The mean and variance of the sample was 347.2 cal and 20.74 cal^2 respectively.

(i) The nutritionist wishes to carry out a hypothesis test on his claim. Explain why t-test instead of z-test is to be used. State an assumption for the test to be valid. [2]

(ii) Test at 5% level of significance, whether the mean number of calories in an energy bar is 350 cal, defining any symbols that you use. [4]

(iii) Suppose the nutritionist uses a different test in part (**ii**). Without further calculation, explain and state whether the conclusion will be different. [2]

The manufacturer of the energy bar refines the manufacturing process and the new energy bars follow a normal distribution with mean μ cal and variance 20.74 cal^2. The manufacturer then provides the nutritionist with another sample of 15 energy bars.

(iv) Find the range of mean number of calories, $\bar{x}$, of the second sample of 15 energy bar so that the null hypothesis in part **(ii)** is not rejected at 5% level of significance. Leave your answer correct to one decimal place. [3]

[Turn over

11. A group of scientists is interested to find out the correlation between the number of species and the size of the natural habitat. The scientists sampled non-overlapping lands of different areas (x) in square kilometres, and noted the corresponding number of species (y) found. The results are shown in the table below.

Area (x)	300	400	500	600	700	800	900	1000	1100	1200
Number of species (y)	12	15	18	21	k	25	26	27	27	28

(i) Given that the equation of the regression line is $y = 0.01758x + 9.018$, show that the value of k to the nearest whole number is 23. [3]

Take k to be 23.

(ii) Draw the scatter diagram for the given data, labelling the axes clearly. [2]

(iii) Calculate the product moment correlation coefficient r. With reference to both the scatter diagram and r, explain why a linear model is not appropriate. [2]

(iv) The following models are suggested for the data.

(A) $y = a + bx^2$, (B) $y = ax^b$, where $a > 0$ and $0 < b < 1$.

Use a graphical approach to determine which model is more appropriate. [2]

(v) Use the more appropriate model to estimate the area of the natural habitat when the number of species found is 24. Comment on the reliability of your estimation. [3]

End of Paper