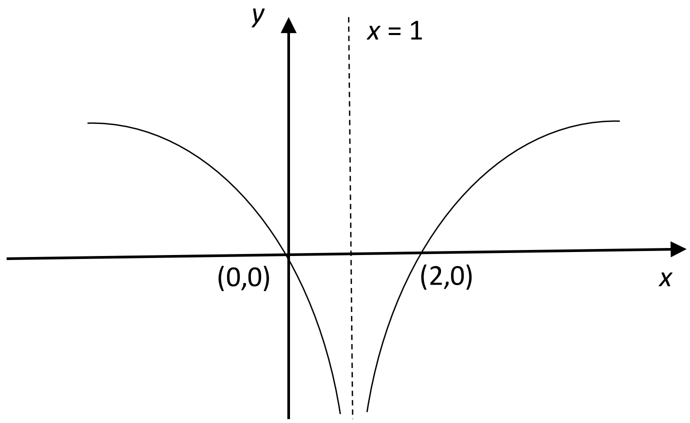
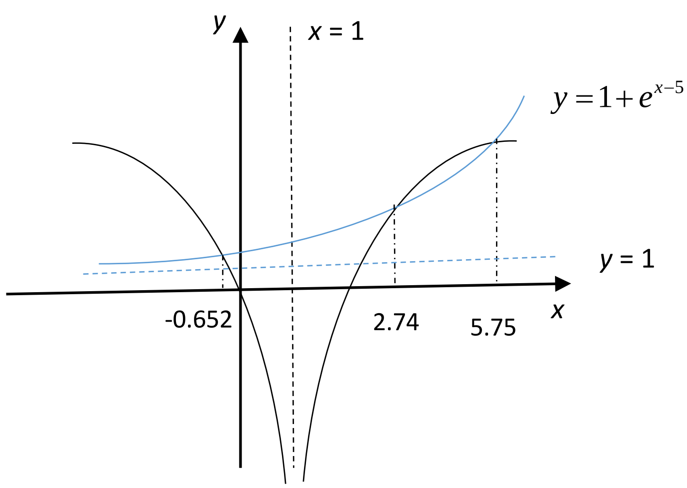
1	$(y-2)x^2+(y+1)x-1=0$ Since there 2 distinct solutions, $b^2-4ac>0$. $\Rightarrow (y+1)^2-4(y-2)(-1)>0 \Rightarrow y^2+2y+1+4y-8>0$ $\Rightarrow y^2+6y-7>0 \Rightarrow (y-1)(y+7)>0$ $\Rightarrow y>1 \text{ or } y<-7, y\neq 2$
2 (i)	
(ii)	$\ln(x-1)^2-1\le e^{x-5}$ $\ln(x-1)^2\le 1+e^{x-5}$ Sketch $y=1+e^{x-5}$.  $-0.652\le x<1 \text{ or } 1<x\le 2.74 \text{ or } x\ge 5.75$

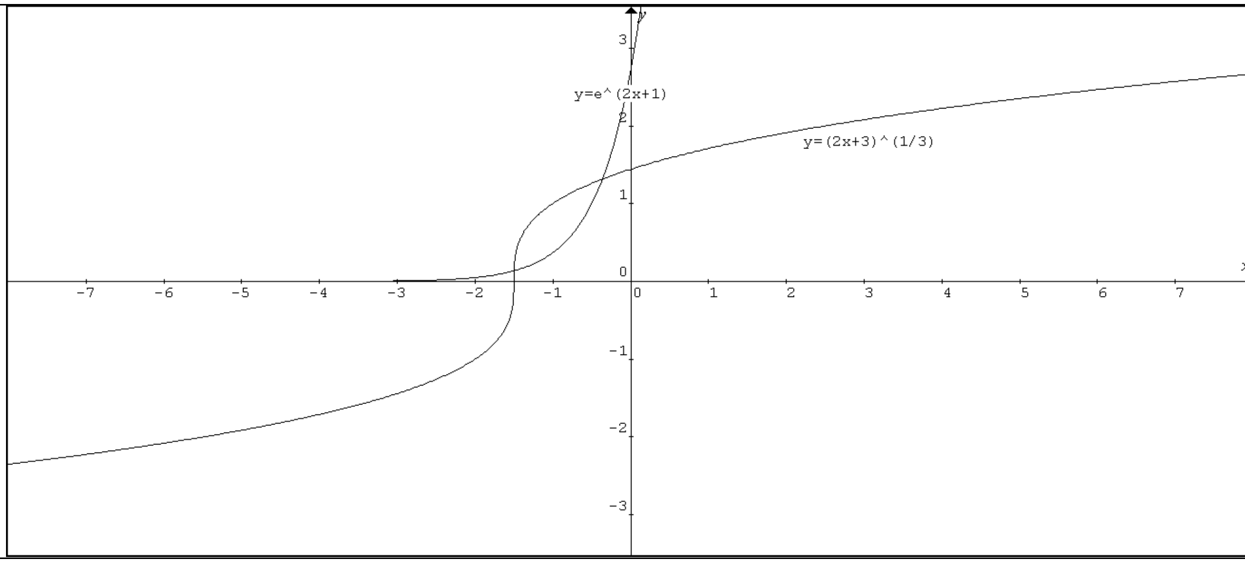
3(i)

$$\int e^{4-3x}\, dx = \frac{-e^{4-3x}}{3} + C$$

(ii)

$$\int_2^3 \left(1+\frac{1}{2x}\right)^2 dx = \int_2^3 \left(1+\frac{1}{x}+\frac{1}{4x^2}\right) dx \quad \left[x + \ln|x| - \frac{1}{4x}\right]_2^3 = \left(3+\ln 3 - \frac{1}{12}\right) - \left(2 + \ln 2 - \frac{1}{8}\right) = \frac{25}{24} + \ln\frac{3}{2}.$$

4(i)



x-coordinates of points of intersection are:
$x = -1.50$ & $x = -0.363$.

(ii)

Area required = $\int_{-1.49875}^{-0.36309} \left(\sqrt[3]{2x+3} - e^{2x+1}\right) dx = 0.531$

5

Volume of cylinder $= \pi r^2 h = \frac{5\pi}{128} \Rightarrow h = \frac{5}{128 r^2}$

Let the cost of the cylinder be C. $\quad C = a(2\pi r^2) + \frac{4a}{5}(2\pi r h)$

$$C = 2a\pi r^2 + \frac{8a\pi}{5} r \left(\frac{5}{128 r^2}\right) \Rightarrow C = 2a\pi r^2 + \frac{\pi a}{16 r}$$

$$\frac{dC}{dr} = 4a\pi r - \frac{\pi a}{16 r^2} = 0 \qquad \Rightarrow r^3 = \frac{1}{64} \Rightarrow r = \frac{1}{4}$$

r	$\left(\frac{1}{4}\right)^-$	$\left(\frac{1}{4}\right)$	$\left(\frac{1}{4}\right)^+$
$\frac{dC}{dr}$	-ve	0	+ve

OR $\quad \frac{d^2C}{dr^2} = 4a\pi + \frac{\pi a}{8r^3} > 0$ when $r = \frac{1}{4}$.

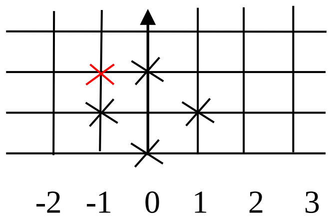
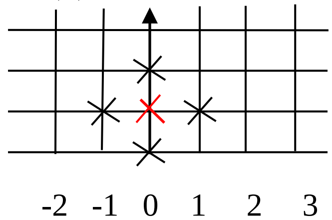
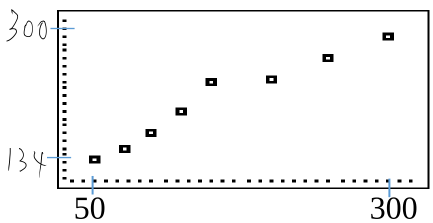
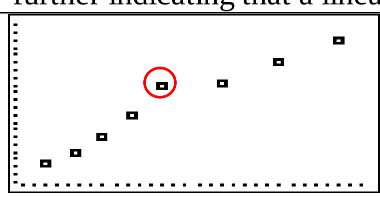
Hence C is minimum when $r = \frac{1}{4}$.

$$h = \frac{5}{128 \times \frac{1}{16}} = \frac{5}{8}$$

The can that costs the least to manufacture has radius $\frac{1}{4}$ m and height $\frac{5}{8}$ m.

6(i)	$y=\frac{1}{6}(2x-3)^3-4x$ $\frac{dy}{dx}=(2x-3)^2-4$ $x=0, \frac{dy}{dx}=5$ Gradient of the normal $=-\frac{1}{5}$
(ii)	when $x=0,\ y=-\frac{27}{6}=-\frac{9}{2}$ Equation of the normal $y+\frac{9}{2}=-\frac{1}{5}x$ $10y+45=-2x$ $2x+10y+45=0$ $y=0,\ x=-\frac{45}{2}$ $\quad N\left(-\frac{45}{2},0\right)$ Area of *OPN* $=\frac{1}{2}\left(\frac{9}{2}\right)\left(\frac{45}{2}\right)=\frac{405}{8}(=50.625)$
(iii)	$(2x-3)^2-4\geq 0$ $(2x-3-2)(2x-3+2)\geq 0$ $(2x-5)(2x-1)\geq 0$ $\Rightarrow\left\{x:\ \text{x}\leq\frac{1}{2}\text{ or } x\geq\frac{5}{2}\right\}$
7(i) (ii)	Number all the students in the school from 1 to 600. Compute the sampling interval, *k*, and obtain $k=\frac{600}{20}=30$. Randomly select an integer from 1 to 30. Select every 30th pupil thereafter until 20 members are obtained. Another alternative method of sampling is stratified sampling. Divide the pupils into various strata such as their education level (Secondary 1 to Secondary 4) and gender (male and female). The sample size for each stratum is proportionally represented. Pupils are then selected randomly within each stratum. <u>OR</u> Another alternative method of sampling is simple random sampling. Number all the students in the school from 1 to 600. Using a random number generator to obtain 20 random numbers. Those students whose numbers correspond to these 20 numbers were those selected. <u>OR</u> Another alternative method of sampling is quota sampling. Divide 600 pupils into male and female. Select 10 males and 10 females based on the teacher's own discretion.

8	$X \sim N(220\,,25)$ $X_1 - X_2 \sim N(0\,,50)$ $P(X_1 - X_2 > 20) = 0.00234$ $Y = bX + c \;\Rightarrow\; \text{Var}(Y) = b^2\,\text{Var}(X) \;\Rightarrow\; 16 = 25\,b^2$ …….. (1) $b^2 = \dfrac{16}{25}$ $\quad b > 0\ \therefore b = \dfrac{4}{5}$ or 0.8 $E(Y) = \dfrac{4}{5}E(X) + c \;\Rightarrow\; 100 = \dfrac{4}{5}(220) + c$…….. (2) $\;\Rightarrow\; c = -76$
9(i)	$P(\text{Mary goes to playground } X \text{ and she chooses a play-house}) = \frac{1}{4} \times \frac{4}{9} = \frac{1}{9}$ $P(\text{Mary goes to playground } Y \text{ and she chooses a play-house}) = \frac{1}{3} \times \frac{2}{12} = \frac{1}{18}$ $P(\text{Mary goes to playground Z and she chooses a play-house}) = \frac{5}{12} \times \frac{1}{16} = \frac{5}{192}$ Probability required $= \frac{1}{9} + \frac{1}{18} + \frac{5}{192} = \frac{37}{192}$ or 0.193 (3 s.f.)
(ii)	$P(\text{Mary goes to playground} Y \mid \text{Mary chooes a climbing frame}) = \dfrac{P(Y \cap C)}{P(C)} = \dfrac{\frac{1}{3} \times \frac{1}{12}}{\frac{1}{3} \times \frac{1}{12} + \frac{5}{12} \times \frac{4}{16}} = \dfrac{4}{19}$ or 0.211 (3 s.f.)
10 (i)	Let X denote the number of days, out of 4 days, Jane woke up late. $X \sim B(4, 0.1)$ Required probability $= [P(X = 2)](0.1)$ $= 0.00486$ (3 s.f.)
(ii)	4 weeks = 20 weekdays Bus fare for 4 weeks = $\$1.50 \times 20 = \$30$. Thus Jane will have to be late at least 4 times. Let Y denote the number of days, out of 20 days, Jane woke up late. $Y \sim B(20, 0.1)$ Required probability $= P(Y \geq 4)$ $= 1 - P(Y \leq 3)$ $= 0.133$ (3 s.f.)
(iii)	Let W be no of days, out of 260 days, Jane woke up late. $W \sim B(260, 0.1)$ $n = 260 > 50$ is large. $\quad np = 234 > 5, nq = 26 > 5$ $W \sim N(26, 23.4)$ approx. Required probability = $P(W > 20)$ $\xrightarrow[cc]{} P(W > 20.5) = 0.872$ (3 s.f.)

11(a)	(i)  (ii) 
(b)(i)	$r = 0.974$
(ii)	
(iii)	$r = 0.974 \approx 1$ There is a strong positive linear correlation between m and l, implying that the linear model is suitable. In addition, the scatter diagram also show that the points lie approximately along a straight line, further indicating that a linear model is suitable.
(iv)	 Outlier: $m = 150, l = 240$
(v)	$r = 0.99567 \approx 0.996$
(vi)	Use the regression line of l on m as m is clearly the controlled variable. $l = 0.66763m + 104.659$ When $l = 250$, $250 = 0.66763m + 104.659 \Rightarrow m = 217.6969 \approx 218$
(vii)	It would not be advisable as $m = 0$ falls outside the given data range of $50 \le m \le 300$. Extrapolation is generally discouraged.
12(i)	$\sum(x - 150) = -35$ $\left(\sum x\right) - 50(150) = -35$ $\sum x = 7465$ $\bar{x} = \frac{7465}{50} = \frac{1493}{10} = 149.3$ $s^2 = \frac{1}{49}(167) = \frac{167}{49} = 3.40816 \approx 3.41(3\text{ sf})$
(ii)	$H_0 : \mu = 150$ against $H_1 : \mu < 150$ Test at 5% significance level, $Z = -2.68$ $p = 0.00367 < 0.05$

	Reject H_0 and conclude that there is sufficient evidence at 5% significance level, the content of the drinks in the bottle is less than what the packaging claimed to be / the complaints are valid.
(iii)	5% significance level means there is a 5% chance that we say that the complaints are valid when the contents are actually not less than 150ml.
(iv) (a)	If the sample was not randomly chosen, the conclusion made can be unreliable. For example, the sample may have been the first 50 bottles that are produced in the same batch and the mean quantity could have changed as production continued.
(iv) (b)	The conclusion is unaffected whether the distribution is normal or not as this is a large sample. By Central Limit Theorem, the mean quantity in the bottled drink is approximately normally distributed.
(v)	$H_0: \mu = 150$ against $H_1: \mu \neq 150$ $p = (0.00367)(2) = 0.00734 < 0.05$ Reject H_0 at 5% significance level that there is sufficient evidence that, on average, the bottled soft drink does not contain 150ml of drinks
13(i)	Let A be the mass of a type A orange in kg. $A \sim \text{N}(0.18\ ,\ 0.05^2)$. Let B be the mass of a type B orange in kg. $B \sim \text{N}(0.20\ ,\ 0.04^2)$. Let $M = B_1 + B_2 + ... + B_5$ $\quad M \sim \text{N}(1\ ,\ 0.008)$ $\text{P}(M \leq 1.1) = 0.868$
(ii)	Let $W = (A_1 + A_2 + ... + A_8) - (B_1 + B_2 + ...B_6)$ $W \sim \text{N}(0.24\ ,\ 0.0296)$ $\text{P}(\lvert W \rvert < 0.3) = \text{P}(-0.3 < W < 0.3)$ $= 0.636$
(iii)	Let C_L be the amount paid by Mr Lim. $C_L = 7(A_1 + A_2 + A_3 + A_4) + 8(B_1 + B_2 + B_3 + B_4)$ $C_L \sim \text{N}(11.44\ ,\ 0.8996)$ Let C_C be the amount paid by Mr Chan. $C_C = 8(B_5 + B_6 + ... + B_{10})$ $\quad C_C \sim \text{N}(9.6\ ,\ 0.6144)$ $C_L - C_C \sim N(1.84\ ,\ 1.514)$ $\quad \text{P}(C_L - C_C \geq 1.8) = 0.513$