2016 SH2 H2 Mathematics Preliminary Examination Paper 2 Suggested Solutions

<table>
<tr><th>Qn No.</th><th>Solution</th></tr>
<tr><td>1 (a)</td><td>

Since Kenny would have paid up his loan in full exactly in the 48th month,

$$S_{48} = \frac{48}{2}\left[2a + (48-1)d\right]$$

$$9600 = 24(2a + 47d)$$

$$400 = 2a + 47d \cdots\cdots (1)$$

Since Kenny had an outstanding payment of \$2400 after the 40th month, total amount paid by the 40th month = \$9600 – 2400 = \$7200. Therefore,

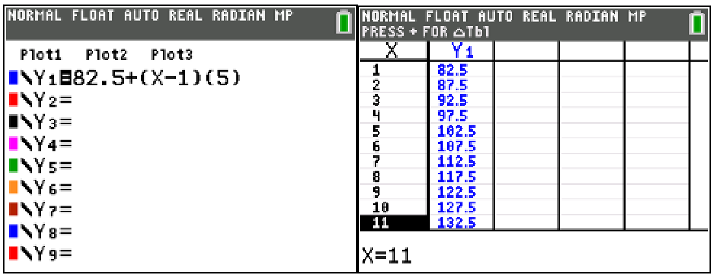
$$S_{40} = \frac{40}{2}\left[2a + (40-1)d\right]$$

$$7200 = 20(2a + 39d)$$

$$360 = 2a + 39d \cdots\cdots (2)$$

Solving (1) & (2), $a = 82.5$, $d = 5$.

On the last day of the n^{th} month (for $1 \le n \le 40$), the amount paid by Kenny = \$82.5 + $(n-1)(5)$.



Therefore,
amount paid on last day of 10th month = \$127.5 < \$130,
amount paid on last day of 11th month = \$132.5 > \$130.

Therefore Kenny first paid at least \$130 on the last day of the **11th** month.

</td></tr>
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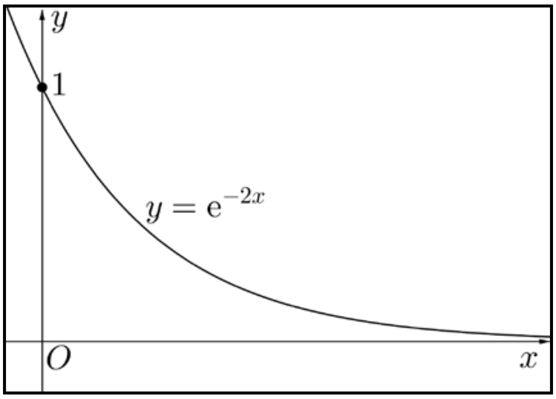
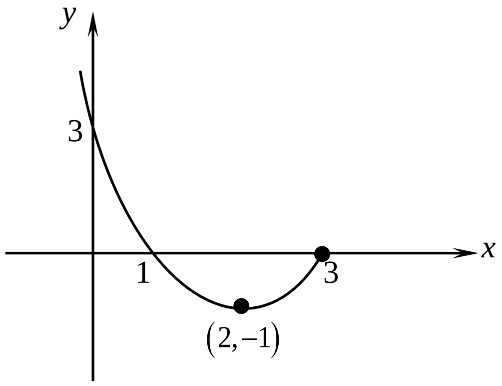
<table>
<tr><th>Qn No.</th><th>Solution</th></tr>
<tr><td>1 (b) (i)</td><td>Common ratio of $1+e^{-2x}+e^{-4x}+\cdots$ is $r=e^{-2x}$.


For $x>0$, $0<e^{-2x}<1$ (see above sketch).
Therefore, the geometric series converges (since e^{-2x} is the common ratio).

OR

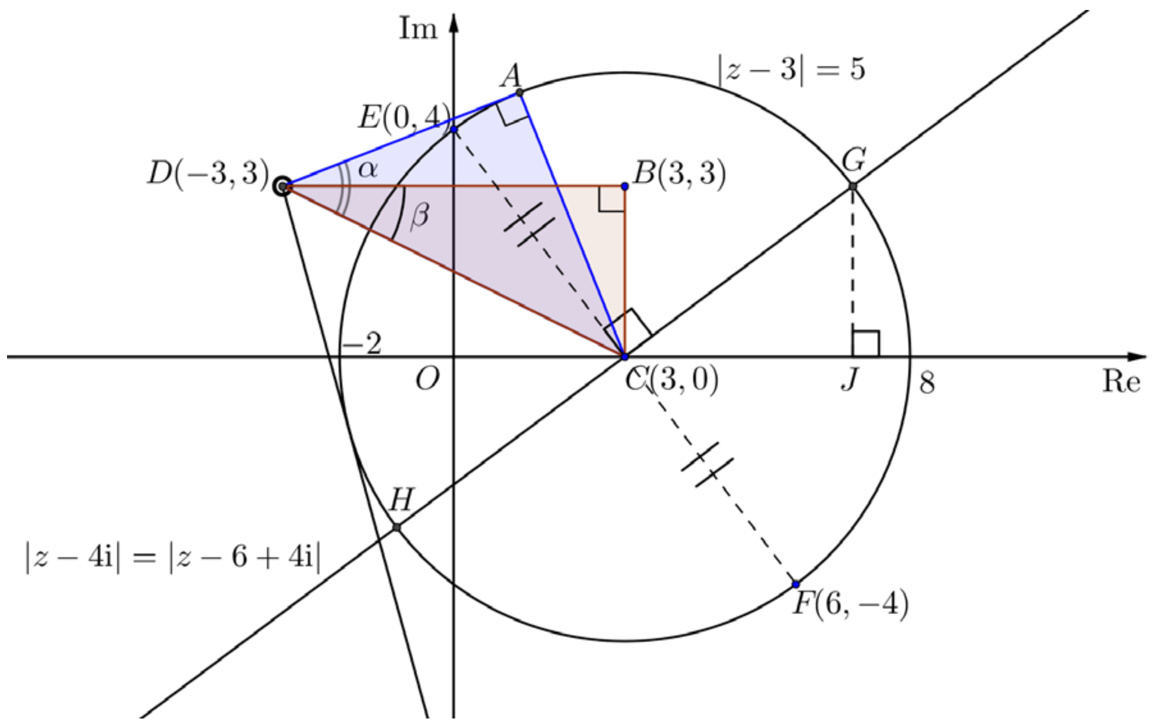
As $n\to\infty$, $e^{-2nx}\to 0$ (for $x>0$). Therefore
$S_n=\frac{1-e^{-2nx}}{1-e^{-2x}}\to\frac{1}{1-e^{-2x}}$, i.e. the series is convergent.
Sum to infinity $=\frac{1}{1-e^{-2x}}$</td></tr>
<tr><td>1 (b) (ii)</td><td>For x = 10, $S_n=\frac{1-e^{-20n}}{1-e^{-20}}$, $S=\frac{1}{1-e^{-20}}$.
$S-S_n<\frac{S}{10^{100}}$
$S_n>S-\frac{S}{10^{100}}$
$S_n>S\left(1-\frac{1}{10^{100}}\right)$
$\frac{1-e^{-20n}}{1-e^{-20}}>\frac{1}{1-e^{-20}}\left(1-\frac{1}{10^{100}}\right)$
$1-e^{-20n}>1-\frac{1}{10^{100}}$
$-e^{-20n}>-\frac{1}{10^{100}}$
$e^{-20n}<\frac{1}{10^{100}}$
$-20n<\ln\frac{1}{10^{100}}=-100\ln 10$
$n>5\ln 10=11.513$
Therefore, least value of n = 12</td></tr>
</table>

Qn No.	Solution
2(a)	$f(x) = x$ $x^2 - 4x + 3 = x$ $x^2 - 5x + 3 = 0$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2}$ $= \frac{5 \pm \sqrt{13}}{2}$ Since $x \le 2$, we have $x = \frac{5 - \sqrt{13}}{2}$
2(b)(i)	 Any horizontal line $y = k$ where $k \in (-1, 0]$ cuts the graph of $y = f(x)$ twice. Thus f is not one-one and hence its inverse does not exist. **OR** The line $y = 0$ (or any appropriate value over $(-1, 0]$ cuts the graph of $y = f(x)$ twice. Thus f is not-one and hence its inverse does not exist. **OR** $f(3) = f(1) = 0$ but $3 \neq 1$. Thus f is not one-one and its inverse does not exist.

Qn No.	Solution
2(b) (ii)	From the graph, $R_f = [-1, \infty)$. Moreover, $D_g = (-2, \infty)$. Since $R_f \subset D_g$, gf exists. **OR** $f(x) = x^2 - 4x + 3 = (x-2)^2 - 1$ For $x \le 3$, $R_f = [-1, \infty)$. Moreover, $D_g = (-2, \infty)$. Since $R_f \subset D_g$, gf exists. $gf(x) = g(x^2 - 4x + 3)$ $\quad = \tan^{-1}(2x^2 - 8x + 7)$ Note that $D_{gf} = D_f$. Thus, $gf : x \mapsto \tan^{-1}(2x^2 - 8x + 7),\ x \in \mathbb{R},\ x \le 3.$ $R_{gf} = \left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$

Qn No.	Solution
3 (i)	Let the foot of perpendicular be N. **Method 1** Equation of the line that passes through A and perpendicular to p_1 is $l_A: \mathbf{r} = \begin{pmatrix} 2q \\ 0 \\ 2 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \gamma \in \mathbb{R}$. Since N lies on l_A, $\overrightarrow{ON} = \begin{pmatrix} 2q \\ 0 \\ 2 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ for some $\gamma \in \mathbb{R}$. $\left[\begin{pmatrix} 2q \\ 0 \\ 2 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right] \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 4 \Rightarrow 2q + 2\gamma = 4$ $\Rightarrow \gamma = 2 - q$ $\therefore \overrightarrow{ON} = \begin{pmatrix} 2q \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2-q \\ 2-q \\ 0 \end{pmatrix} = \begin{pmatrix} 2+q \\ 2-q \\ 2 \end{pmatrix}$ Hence, N is the point $(2+q, 2-q, 2)$. **Method 2** Let C denote the point (0, 4, 2). Then C lies on p_1 since LHS of eqn. of p_1 = 0 + 4 = 4 = RHS of eqn. of p_1 $\overrightarrow{AC} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2q \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -2q \\ 4 \\ 0 \end{pmatrix}$ $\overrightarrow{AN}$ = Proj. vector of $\overrightarrow{AC}$ onto $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $= \dfrac{\begin{pmatrix} -2q \\ 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{1^2+1^2+0^2}} \left(\dfrac{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{1^2+1^2+0^2}} \right)$ $= \dfrac{4-2q}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = (2-q) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\therefore \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN} = \begin{pmatrix} 2q \\ 0 \\ 2 \end{pmatrix} + (2-q) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2+q \\ 2-q \\ 2 \end{pmatrix} \gamma$ Hence, N is the point $(2+q, 2-q, 2)$.

Qn No.	Solution
3 (ii)	Let **b** be the position vector of point *B*. By Ratio Theorem, $\begin{pmatrix}2q\\0\\2\end{pmatrix}+\mathbf{b}=2\begin{pmatrix}2+q\\2-q\\0\end{pmatrix}$ $\mathbf{b}=\begin{pmatrix}4\\4-2q\\2\end{pmatrix}$ Since *B* lies in p_2, $\begin{pmatrix}4\\4-2q\\2\end{pmatrix}\cdot\begin{pmatrix}3\\2\\-5\end{pmatrix}=7$ $12+8-4q-10=7$ $q=0.75$ **or** $\frac{3}{4}$
3 (iii)	Using GC, $l:\mathbf{r}=\begin{pmatrix}-1\\5\\0\end{pmatrix}+\theta\begin{pmatrix}5\\-5\\1\end{pmatrix},\theta\in\mathbb{R}$.
3 (iv)	$\begin{pmatrix}\lambda\\0\\1\end{pmatrix}\cdot\begin{pmatrix}5\\-5\\1\end{pmatrix}=0\Rightarrow\lambda=-\frac{1}{5}.$ $\begin{pmatrix}-1\\5\\0\end{pmatrix}\cdot\begin{pmatrix}\lambda\\0\\1\end{pmatrix}\neq\mu\Rightarrow\mu\neq\frac{1}{5}.$

<table>
<tr><th>Qn No.</th><th>Solution</th></tr>
<tr><td>4 (i), (iii) (1st part)</td><td>


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<tr><td>4 (ii)</td><td>

$CD = \sqrt{(3-(-3))^2 + (0-3)^2}$

$\quad = \sqrt{45}$

Largest value of $\arg(z+3-3i)$
$= \alpha - \beta$

$= \sin^{-1}\left(\frac{AC}{CD}\right) - \tan^{-1}\left(\frac{BC}{BD}\right)$

$= \sin^{-1}\left(\frac{5}{\sqrt{45}}\right) - \tan^{-1}\left(\frac{1}{2}\right)$

$= 0.37742$

$= 0.377$ rad (to 3 s.f.)
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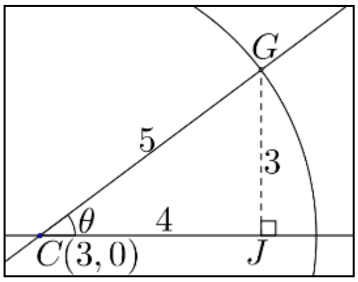
<tr><td>4 (iii) (2nd part)</td><td>

<u>Method 1: Applying relationship between gradient of a line and the angle it makes with the positive horizontal axis</u>

Gradient of perpendicular bisector $= -\frac{1}{\left(-\frac{4}{3}\right)} = \frac{3}{4}$

Angle that perpendicular bisector makes with positive real axis, $\theta = \tan^{-1}\left(\frac{3}{4}\right)$

Hence,
$\frac{GJ}{5} = \sin\theta = \frac{3}{5} \Rightarrow GJ = 3$
$\frac{CJ}{5} = \cos\theta = \frac{4}{5} \Rightarrow CJ = 4$
So G and H represent the complex numbers
$z = (3 + 4) + (0 + 3)i = 7 + 3i$ (corresponding to G), and
$z = (3 – 4) + (0 – 3)i = –1 – 3i$ (corresponding to H) resp.



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<tr><td>4 (iii) (2nd part)</td><td>

<u>Method 2: Using Similar Triangles</u>

$\angle GCE = 90^\circ \Rightarrow \angle OCE + 90^\circ + \angle GCJ = 180^\circ$

$\Rightarrow \angle OCE = 90^\circ - \angle GCJ$

Also, $\angle OEC = 90^\circ - \angle OCE$

$= 90^\circ - \left(90^\circ - \angle GCJ\right)$

$= \angle GCJ$

Furthermore, $\angle COE = \angle GJC = 90^\circ$.

Therefore, $\triangle COE \sim \triangle GJC$. Hence,

$\frac{CO}{CE} = \frac{GJ}{GC} \Rightarrow \frac{3}{5} = \frac{GJ}{5} \Rightarrow GJ = 3$, and

$\frac{OE}{CE} = \frac{CJ}{GC} \Rightarrow \frac{4}{5} = \frac{CJ}{5} \Rightarrow CJ = 4$

So coordinates of *G* are (3 + 4, 0 + 3), i.e. (7, 3) and similarly, coordinates of *H* are (3 – 4, 0 – 3), i.e. (–1, –3). Therefore, possible values of *z* are 7 + 3i and –1 –3i.

<u>Method 3: Using Cartesian Equations</u>

Equation of circle: $\left(x-3\right)^2 + y^2 = 5^2$

Equation of perpendicular bisector:

$y - 0 = -\frac{1}{\left(-\frac{8}{6}\right)}(x-3) \Rightarrow y = \frac{3}{4}(x-3)$

Substituting, $\left(x-3\right)^2 + \left(\frac{3}{4}(x-3)\right)^2 = 5^2$

$\left(1 + \left(\frac{3}{4}\right)^2\right)\left(x-3\right)^2 = 5^2$

$\frac{25}{16}\left(x-3\right)^2 = 25$

$\left(x-3\right)^2 = 16$

$x - 3 = \pm 4$

$x = 7$ or -1.

When $x = 7$, $y = \frac{3}{4}(7-3) = 3$

When $x = -1$, $y = \frac{3}{4}(-1-3) = -3$

Therefore, possible values of *z* are 7 + 3i and –1 –3i.

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Qn No.	Solution
5 (a)	Assign a number from 1 to N to each of the students, where N represents the student population size OR obtain a list of the students from the administration office in order of their identification numbers or registration numbers. Next, determine the sampling interval size $k = \frac{1}{0.02} = 50$. Randomly select any student from the list, say the 1st student. Select every 50th student thereafter (i.e. 51st, 101th,…) until the required sample is obtained.
5 (b)	**Advantages**: • **<u>Representativeness of Sample</u>** Quota sampling allows the survey to capture the responses that represent various groups of students (e.g. different PM classes, or **1st** CCAs); this may be preferred as certain homeroom or sports facilities may not be in as good a condition as others, and the representation of each group will ensure that the results will not be biased towards those who are often using these less functional facilities or towards those who are often using the more functional facilities. • **<u>Efficiency of Collecting the Sample</u>** Quota sampling may be more efficient as systematic sampling in this case requires the surveyor to identify the selected respondents and to contact them, which can be time consuming (e.g. student selected may be on MC on day of survey, selected students do not respond to online survey etc). **Disadvantages**: • **<u>Non-randomness/Selection Bias</u>** Quota sampling is non-random and may contain selection bias, where the surveyor chooses people who may appear friendlier or choose students in the canteen only at a selected time period. This results in certain students having no chance of being selected at all, which may affect the validity of the survey results. • **<u>Non-representativeness of Sample</u>** Quota sampling may result in a group (e.g. one entire cohort, or people coming later to the canteen etc.) being excluded entirely from the selection, which may result in the data collected being an inaccurate representation of the entire school population.

Qn No.	Solution
6	$X \sim N(\mu, \sigma^2)$ $P(X < 17.7) = 0.15$ $P\left(Z < \frac{17.7-\mu}{\sigma}\right) = 0.15$ $\frac{17.7-\mu}{\sigma} = -1.03643$ --- (1) $P(X > 21.9) = 0.2$ $P(X < 21.9) = 0.8$ $P\left(Z < \frac{21.9-\mu}{\sigma}\right) = 0.8$ $\frac{21.9-\mu}{\sigma} = 0.841621$ --- (2) Solving simultaneous equations (1) and (2): From (1): $\mu = 1.03643\sigma + 17.7$ From (2): $\mu = -0.841621\sigma + 21.9$ Using GC, $\mu = 20.0$ (3s.f) and $\sigma = 2.24$ (3s.f)

<table>
<tr><th>Qn No.</th><th>Solution</th></tr>
<tr><td>7 (i)</td><td>

No. of ways $=\left(\frac{9!}{9}\right)\binom{9}{6}6! = 2\,438\,553\,600$

</td></tr>
<tr><td>7 (ii)</td><td>

No. of ways

$=\left(\frac{3!}{3}\right)(5!)(4!)(4!)(1)$

$=138\,240$

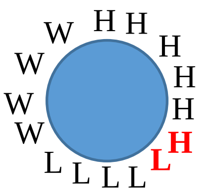
OR

<u>Case 1</u>

No. of ways

$=(5!)(4!)(4!)$

$=138\,240/2$

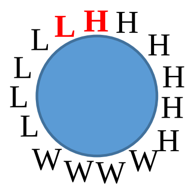


<u>Case 2</u>

No. of ways

$=(5!)(4!)(4!)$

$=138\,240/2$



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<tr><td>7 (last part)</td><td>

<u>Case 1</u> : None from Liaison Committee

No of ways $=1$

<u>Case 2</u> : None from Welfare Committee

No of ways $=\binom{6}{6}\binom{5}{4}+\binom{6}{5}\binom{5}{5}=11$

or

No of ways $=\binom{11}{10}=11$

Total Number of ways to select at least 3 men and 3 women

$=\binom{15}{10}-1-11$

$=2991$

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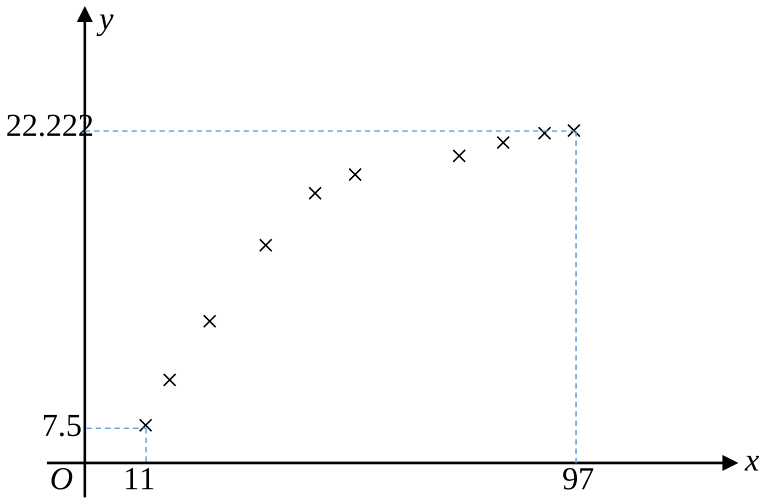
Qn No.	Solution
8 (i)	
8 (ii)	**<u>Unsuitability of a Linear Model</u>** A linear model predicts the average diameter will keep increasing indefinitely without any limit. Therefore a linear model is not appropriate. **<u>Unsuitability of a Quadratic Model</u>** A quadratic model predicts that the average diameter will eventually attain a maximum value, and thereafter decrease as the age increases, till it eventually takes on negative values. This is not possible, and therefore a quadratic model is not appropriate.
8 (iii)	Using the suggested model, the least square regression line is $y = 23.886 - \frac{185.346}{x} = 23.9 - \frac{185}{x}$ (to 3 s.f.) r-value = –0.994 (to 3 s.f.) When $x = 40$, $y = 23.886 - \frac{185.346}{40} = 19.3$ (to 3 s.f.) Since x = 40 is within the range of values of x, [11, 97] and the product moment correlation coefficient, –0.994, has an absolute value that is close to 1, suggesting a strong linear correlation between the variables y and $1/x$, therefore the estimate is reliable.

Qn No.	Solution
9 (i)	The eye colour of a person is independent of that of another person. **OR** Every person is equally likely to have blue eyes.
9 (ii)	$Y \sim \mathrm{B}\left(60, 0.08\right)$ Then $\mathrm{P}(5 \le Y < 21)$ $= \mathrm{P}(Y \le 20) - \mathrm{P}(Y \le 4)$ $= 0.530$ (to 3 s.f.)
9 (iii)	Since $n = 60$ (> 50) is large and $np = 4.8 < 5$, $Y \sim \mathrm{Po}(4.8)$ approximately. $\mathrm{P}(Y > 9) = 1 - \mathrm{P}(Y \le 9) = 0.0251$ (to 3 s.f.)

Qn No.	Solution
10 (i) (a)	Let E and F denote the event that the sum of scores is at least 8 and the event that the absolute difference between the scores is at most 4 respectively.

Sum of Scores

1st die \ 2nd die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

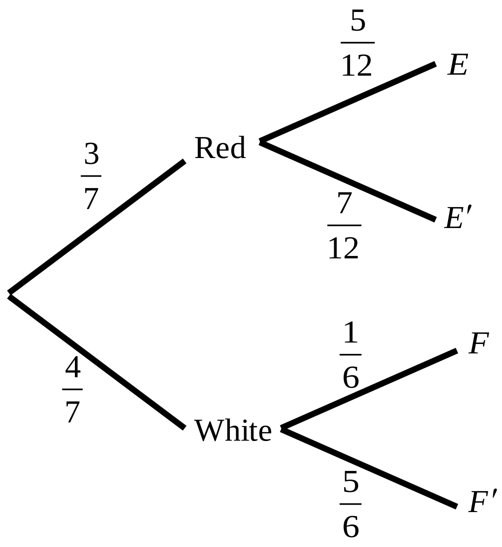
$$\mathrm{P}(E)=\frac{1}{6}\times\frac{1}{6}\times 15=\frac{5}{12}$$

10 (i) (b)

Absolute Difference between Scores

1st die \ 2nd die	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

$$\mathrm{P}(F)=\frac{1}{6}\times\frac{1}{6}\times 6=\frac{1}{6}$$

Qn No.	Solution
10 (ii)	 $\text{P}(\text{game ends at 1st round})$ $= \text{P}(\text{red and game ends at 1st round})$ $+\text{P}(\text{white and game ends at 1st round})$ $= \left(\frac{3}{7}\right)\left(\frac{5}{12}\right)+\left(\frac{4}{7}\right)\left(\frac{1}{6}\right)$ $= \frac{23}{84}$
10 (iii)	P(red ball selected | game ends at the first round) $= \frac{\text{P(red ball selected } \cap \text{game ends at the first round)}}{\text{P(game ends at the first round)}}$ $= \frac{\left(\frac{3}{7}\right)\left(\frac{5}{12}\right)}{\frac{23}{84}} = \frac{15}{23}$
10 (iv)	P(total of 3 rounds of game & exactly 2 white balls selected) $= \text{P(WWR)} + \text{P(WRW)} + \text{P(RWW)}$ $= \left(\frac{4}{7}\right)\left(\frac{5}{6}\right)\left(\frac{3}{6}\right)\left(\frac{5}{6}\right)\left(\frac{3}{5}\right)\left(\frac{5}{12}\right)+\left(\frac{4}{7}\right)\left(\frac{5}{6}\right)\left(\frac{3}{6}\right)\left(\frac{7}{12}\right)\left(\frac{3}{5}\right)\left(\frac{1}{6}\right)$ $+\left(\frac{3}{7}\right)\left(\frac{7}{12}\right)\left(\frac{4}{6}\right)\left(\frac{5}{6}\right)\left(\frac{3}{5}\right)\left(\frac{1}{6}\right)$ $= \frac{25}{504}+\frac{1}{72}+\frac{1}{72}$ $= \frac{13}{168}$

<table>
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<tr><td>11 (i)</td><td>Unbiased estimate of μ, $\bar{x}=\frac{\sum x}{n}=\frac{21350}{20}=1067.5$
Unbiased estimate of σ^2, $s^2=\frac{1}{19}(345900)$
$=18205.3$ (to 1 d.p.)</td></tr>
<tr><td>11 (ii)</td><td>H_0: $\mu=1000$
H_1: $\mu>1000$

Assume that the amounts of loans borrowed by the bank's clients follow a normal distribution.

OR

Assume that the amount of loans borrowed by each client follows a normal distribution.

Level of Significance: 5% (upper-tailed)

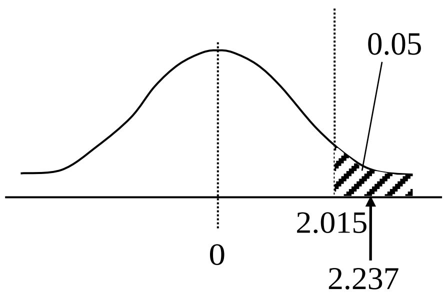
Under H_0, $T=\frac{\bar{X}-1000}{S/\sqrt{20}}\sim t(19)$
Test Statistic: $t=\frac{\bar{x}-1000}{s/\sqrt{20}}$

<u>Method 1: Using critical region and observed test statistic, t</u>

Critical region: $t>2.015$

$t=\frac{1067.5-1000}{s/\sqrt{20}}\approx 2.237$ $\left(s=\sqrt{18205.3}\right)$

Since $t=2.237>2.015$, we reject H_0.



<u>Method 2: Using p-value</u>

p-value = 0.0187

Since p-value = 0.0187 < 0.05, we reject H_0.

We conclude that there is sufficient evidence at 5% level of significance that the company has understated the mean amount of loans borrowed by its clients.</td></tr>
<tr><td>11 (iii)</td><td>The meaning of 'at the 5% significance level' is that there is a probability of 0.05 that it was wrongly concluded that the company had understated the mean amount of loans borrowed by its clients.</td></tr>
</table>

Qn No.	Solution
11 (iv)	Test Statistic: $z=\dfrac{\overline{x}-1000}{250/\sqrt{n}}$ To not reject H_0, $z\le 1.6449$ $\dfrac{1067.5-1000}{250/\sqrt{n}}\le 1.6449$ $\sqrt{n}\le 6.092$ $n\le 37.1$ Since $n\in\mathbb{Z}^+$, $n\le 37$.

Qn No.	Solution
12 (i)	Any one of the following: [*Constant mean rate*] The mean number of cars joining the immigration checkpoint queue for any subinterval of the same length of time within 1 hour (e.g. minute) is constant. **OR** [*Independence of occurrence of event*] Cars join the immigration queue independently of one another, throughout the entire hour.
12 (ii)	Let X denote the random variable for the number of cars leaving an immigration checkpoint queue in a period of n minutes. $X \sim \text{Po}\left(\frac{27}{60}n\right)$ i.e. $X \sim \text{Po}(0.45n)$ $\text{P}(X \geq 1) > 0.9$ $1 - \text{P}(X = 0) > 0.9$ $\text{P}(X = 0) < 0.1$ $\frac{\text{e}^{-0.45n}(0.45n)^0}{0!} < 0.1$ $\text{e}^{-0.45n} < 0.1$ $-0.45n < \ln(0.1)$ $n > \frac{\ln(0.1)}{-0.45} \approx 5.11$ Therefore, least integer n is 6.

Qn No.	Solution
12 (iii)	Let J and L denote the random variables for the number of cars joining and leaving an immigration checkpoint queue respectively in a 2-hour period. Then $$J \sim \text{Po}(46) \text{ and } L \sim \text{Po}(54).$$ Since $46 > 10$, $J \sim \text{N}(46, 46)$ **approximately**. Since $54 > 10$, $L \sim \text{N}(54, 54)$ **approximately**. Let W denote the number of people in the queue at 1100. Then $W = 19 + J - L$. $\text{E}(W) = \text{E}(19 + J - L)$ $= 19 + \text{E}(J) - \text{E}(L) = 19 + 46 - 54 = 11$, and $\text{Var}(W) = \text{Var}(19 + J - L)$ $= \text{Var}(J) + \text{Var}(L) = 46 + 54 = 100$ Therefore, $W = 19 + J - L \sim \text{N}(11, 100)$ **approximately.** $\text{P}(19 + J - L \leq 12) = \text{P}(19 + J - L \leq 12.5)$ (by c.c.) $= 0.560$ (to 3.s.f) **<u>Alternatively,</u>** $J - L \sim \text{N}(-8, 100)$ **approximately.** $\text{P}(J - L \leq -7) = \text{P}(J - L \leq -6.5)$ $= 0.560$ (to 3.s.f) **<u>Or equivalently,</u>** $L - J \sim \text{N}(8, 100)$ **approximately.** $\text{P}(L - J \geq 7) = \text{P}(L - J \geq 6.5)$ $= 0.560$ (to 3.s.f)
12 (iv)	The mean number of cars joining the immigration checkpoint queue every hour may not be constant due to peak periods as there may be more cars heading to or returning from work.