NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS — 9740/01

Paper 1 — **25 August 2016**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

National Junior College

1 A local café, Toast Rox, sells its coffee in three sizes (regular, medium and large). Toast Rox customers get a 12.5% discount on their total bill if they buy at least 12 cups of coffee, regardless of size. The number of cups of coffee bought by three particular customers and the total amount they paid are shown in the following table.

Customer	Regular	Medium	Large	Amount paid
A	5	3	2	$20.90
B	3	4	1	$17.10
C	2	8	4	$28.00

Find the original price of each of the 3 sizes of coffee drink. [3]

2 **(i)** By using an algebraic method, solve the inequality

$$\frac{3x^2+14}{(x+1)(x+2)} \geq 2.$$ [4]

(ii) Hence, showing all your working clearly, solve the inequality

$$\frac{3x^2+14}{(|x|-1)(|x|-2)} \geq 2.$$ [2]

3 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. Point C lies on the line segment AB, such that AC: CB = 2 : 1. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vector of C. [1]

If the angle between $\mathbf{a}$ and $\mathbf{b}$ is 60°, show that the length of projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is

$$\frac{1}{3}\left(|\mathbf{a}|+|\mathbf{b}|\right).$$ [4]

4 **(a)** Two complex numbers z and w are such that

$$2w - z = 6\mathrm{i} \quad \text{and} \quad wz = \frac{13}{2}.$$

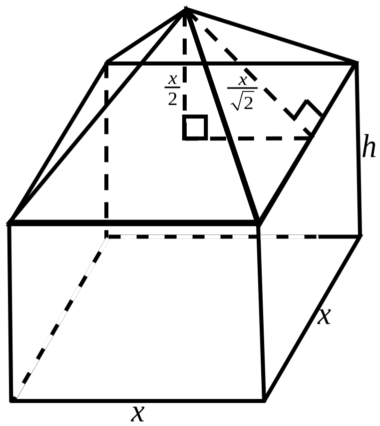
Find w and z, giving each answer in the form $a + b\mathrm{i}$, where a and b are real numbers. [4]

(b) The points P and Q represent the fixed complex numbers p and q respectively. It is given that $0 < \arg p < \arg q < \frac{\pi}{2}$, $|p| = 1$, $|q| = 2$, and $\arg q = 2\arg p$.

In a single Argand diagram, sketch and label the points P, Q, and the points R and S representing q^* and $q^* + 2p^2$ respectively, showing clearly any geometrical relationships. Identify the shape of the quadrilateral $OQSR$, where O is the origin. [4]

[Turn over

5 [It is given that volume of pyramid $= \frac{1}{3} \times \text{base area} \times \text{height}$.]



A model of a house is made up of the following parts.

- The roof is modelled by a pyramid with a square base of sides x cm and height $\frac{x}{2}$ cm. For each triangular side of the prism, the length of the perpendicular from the vertex to the base is $\frac{x}{\sqrt{2}}$ cm.
- The walls are modelled by rectangles with sides x cm and h cm as shown in the diagram.
- The base is a square with sides x cm.

All the parts are joined together as shown in the diagram. The model is made of material of negligible thickness. It is given that the volume of the model is a fixed value V cm^3 and the external surface area is at a minimum value, A cm^2. Use differentiation to find

(i) x, in the form $pV^{\frac{1}{3}}$, and

(ii) A, in the form $qV^{\frac{2}{3}}$,

leaving the values of p and q correct to 3 decimal places. [6]

6 A curve C has parametric equations

$$x = t^3 - kt, \qquad y = 3\left(t^2 - k\right),$$

where k is a positive constant and t is a real parameter.

(i) Sketch C, labelling clearly the coordinates of any points of intersection with the axes. [2]

(ii) Find $\frac{dy}{dx}$ in terms of t and k. [2]

(iii) Find, in terms of k, the exact equation of the tangent to C at the point where $t = -\sqrt{\frac{k}{3}}$. [3]

(iv) Given that the tangent found in part **(iii)** intersects C again at the point $\left(\frac{2}{3}k, k\right)$, find the value of k. [2]

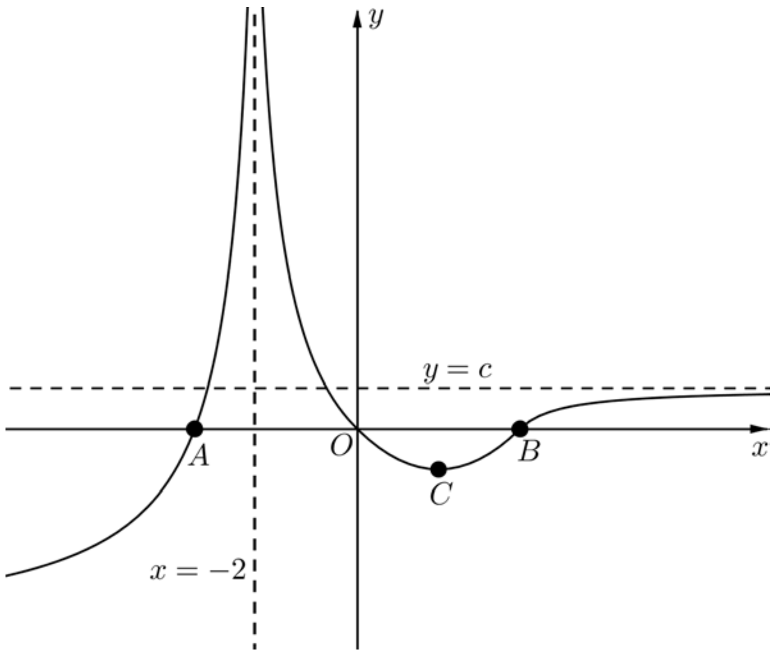
[Turn over

7 **(i)** Given that $y = \ln(\sec x)$, show that $\dfrac{d^3y}{dx^3} = 2\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)$. [2]

(ii) Hence, by further differentiation, find the first two non-zero terms in the Maclaurin's series for y. [3]

(iii) The equation $\dfrac{1}{12}x^2 + \ln(\sec x) = \cos 2x$ has a positive root α close to zero. Use the result in part **(ii)** and the first three terms of the Maclaurin series for $\cos 2x$ to obtain an approximation to α, leaving your answer in surd form. [3]

8 The diagram below shows the curve with equation $y = f(x)$. The curve passes through the origin O, crosses the x-axis at the points A and B, and has a turning point at C. The coordinates of A, B and C are $(-4, 0)$, $(4, 0)$ and $\left(a, -\dfrac{b}{2}\right)$ respectively, where a and b are positive constants such that $a > 1$. The curve also has asymptotes $x = -2$ and $y = c$, where $c > 1$.



On separate diagrams, sketch the following curves, labelling clearly any asymptotes, axial intercepts and turning points in terms of a, b and c whenever necessary.

(a) $y = f(1-2x)$ [3]

(b) $y^2 = f(x)$ [3]

(c) $y = \dfrac{1}{f(x)}$ [4]

9 The gradient of a curve at the point (x, y) is given by the differential equation

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} - 1 = \frac{x-2}{y}.$$

(i) By using the substitution $y = z - x$, find the equation of the curve such that it has a minimum point at $(1, 1)$. [6]

(ii) Sketch the curve, indicating clearly the axial intercept(s) and the minimum point. [2]

10 **(a)** Using partial fractions, find $\displaystyle\int \frac{5x^2 - 2x + 7}{(1-x)\left(2x^2+3\right)}\,\mathrm{d}x$. [6]

(b) **(i)** Differentiate $\sin\left(\mathrm{e}^{-x}\right)$ with respect to x. [1]

(ii) Obtain a formula for $\int_0^n \mathrm{e}^{-2x}\cos\left(\mathrm{e}^{-x}\right)\mathrm{d}x$ in terms of n, where $n > 0$. [3]

(iii) Hence find $\int_0^\infty \mathrm{e}^{-2x}\cos\left(\mathrm{e}^{-x}\right)\mathrm{d}x$ exactly. [2]

11 **(a)** Prove by the method of mathematical induction that

$$\frac{2}{1^2 \times 3^2} + \frac{3}{2^2 \times 4^2} + \cdots + \frac{n+1}{n^2(n+2)^2} = \frac{5}{16} - \frac{1}{4(n+1)^2} - \frac{1}{4(n+2)^2}.$$ [5]

(b) **(i)** By expressing $\dfrac{4n+5}{n(n+1)}$ in partial fractions, show that

$$\sum_{n=1}^{N}\left[\frac{4n+5}{n(n+1)}\left(\frac{1}{5^{n+1}}\right)\right] = a + \frac{b}{(N+1)5^{N+1}},$$

for some real constants a and b to be determined exactly. [3]

(ii) State the sum to infinity of the series in part **(b)(i)**. [1]

(iii) Use your answer to part **(b)(i)** to find $\displaystyle\sum_{n=2}^{N-2}\left[\frac{4n+1}{n(n-1)}\left(\frac{1}{5^n}\right)\right]$ in terms of N. [2]

12 The curves C_1 and C_2 have equations $x^2+16(y-1)^2=16$ and $x^2-16(y-1)^2=16$ respectively.

(i) Verify that the point $(4,1)$ lies on both C_1 and C_2. [1]

(ii) Sketch C_1 and C_2 on the same diagram, labelling clearly any points of intersection with the axes and the equations of any asymptotes. [4]

(iii) The region *R* is bounded by the two curves C_1, C_2 and the positive *x*-axis. Find the numerical value of the volume of revolution formed when *R* is rotated completely about the *x*-axis. [3]

S is the region bounded by C_1.

(iv) Using the substitution $y=1+\cos\theta$, where $-\pi<\theta\leq\pi$, evaluate $\int_0^2 \sqrt{1-(y-1)^2}\,\mathrm{d}y$ exactly. [4]

(v) Hence find the exact area of *S*. [2]

– END OF PAPER –