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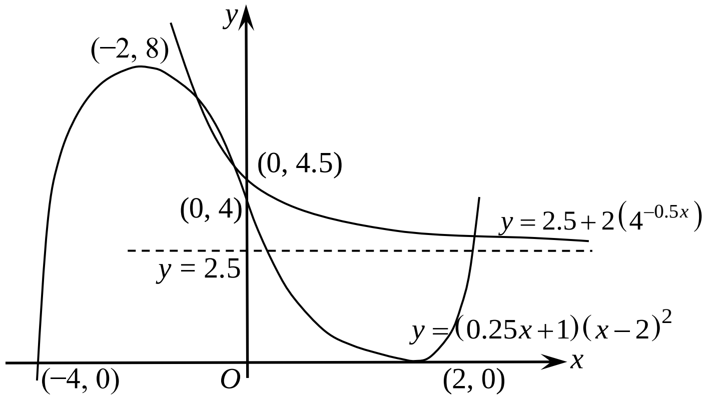
1 (a)	$\frac{d(7\ln(3-4x))}{dx}=\frac{7}{3-4x}.(-4)=\frac{-28}{3-4x}$ or $\frac{28}{4x-3}$
1 (b)	$\int_0^2 3-e^{2x}\,dx=\left[3x-\frac{1}{2}e^{2x}\right]_0^2=\left(6-\frac{1}{2}e^4\right)-\left(0-\frac{1}{2}.1\right)=6\frac{1}{2}-\frac{1}{2}e^4$
2	 $(0.25x+1)(x-2)^2>2.5+2(4^{-0.5x})\Rightarrow -1.25<x<-0.359$ or $x>3.23$ (3 sf) Note: intersections at (−1.25, 7.27), (−0.359, 5.06) & (3.23, 2.71)
3	Given $x-2=10^y$, we have $x = 10^y + 2$ … (1) Substitute (1) into $2x+1=10^{2y}$ $\Rightarrow 2(10^y+2)+1=10^{2y}$ $\Rightarrow 10^{2y}-2(10^y)-5=0$ $\Rightarrow 10^y=\frac{2\pm\sqrt{4-4(1)(-5)}}{2(1)}$ $\Rightarrow 10^y=\frac{2\pm\sqrt{24}}{2}$ Since $10^y > 0$, $10^y=\frac{2+\sqrt{24}}{2}$ $y=\log\left(\frac{2+\sqrt{24}}{2}\right)$ or $y=\log(1+\sqrt{6})$ and $x=10^y+2=\frac{2+\sqrt{24}}{2}+2=(1+\sqrt{6})+2$ giving $x=3+\sqrt{6}$ <u>Alternatively</u> $(x-2)^2 = 2x + 1$ $x^2 - 6x + 3 = 0$ $\Rightarrow x=\frac{6\pm\sqrt{36-4(3)}}{2}=\frac{6\pm\sqrt{24}}{2}$ $\Rightarrow 10^y=\frac{6\pm\sqrt{24}}{2}-2=1\pm\frac{\sqrt{24}}{2}$ Since $10^y > 0$, $10^y=1+\frac{\sqrt{24}}{2}$ giving $y=\log\left(1+\frac{\sqrt{24}}{2}\right)$ or $y=\log(1+\sqrt{6})$ and $x=3+\frac{\sqrt{24}}{2}$ or $x=3+\sqrt{6}$
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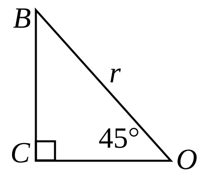
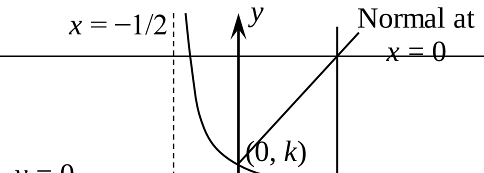
4 (i)	$2\,BC^2 = r^2$ $BC = \dfrac{r}{\sqrt{2}} = \dfrac{r\sqrt{2}}{2}$ (shown)  Alternatively: $\sin 45° = BC \div r \Rightarrow BC = r \sin 45° = \dfrac{r}{\sqrt{2}} = \dfrac{r\sqrt{2}}{2}$ Or use $\cos 45° = BC \div r$ since angle OBC = angle BOC = 45°
4 (ii)	Area = $BOC + AOB = \dfrac{1}{2}\left(\dfrac{r\sqrt{2}}{2}\right)^2 + \dfrac{3}{8}\pi r^2$ $= \dfrac{1}{2}\left(\dfrac{r^2}{2}\right) + \dfrac{3}{8}\pi r^2 = \dfrac{1}{8}(2+3\pi)r^2$ (shown)
4 (last part)	Volume $= \dfrac{1}{8}(2+3\pi)r^2 \times CD = (2+3\pi)5 \quad \Rightarrow \quad CD = \dfrac{40}{r^2}$ Total Area = A = roof + 2 (area of AOB) $= \dfrac{3}{8}(2\pi r)\times CD + \dfrac{3}{8}(\pi r^2)\times 2$ $= \dfrac{3}{8}(2\pi r)\times \dfrac{40}{r^2} + \dfrac{3}{8}(\pi r^2)\times 2 = \dfrac{30\pi}{r} + \dfrac{3\pi r^2}{4}$ $\dfrac{dA}{dr} = -\dfrac{30\pi}{r^2} + \dfrac{3\pi r}{2} = 0 \quad \Rightarrow \quad \dfrac{30\pi}{r^2} = \dfrac{3\pi r}{2} \quad \Rightarrow \quad r^3 = 20$ $\Rightarrow \quad r = (20)^{1/3}$ or r = 2.71 (3 sf) Using First Derivative Test, $\dfrac{dA}{dr} = \dfrac{-60\pi + 3\pi r^3}{2r^2}$ r: 2.7, 2.71, 2.75 sign of $\dfrac{dA}{dr}$: −, 0, + \ — / $\therefore$ total area is a minimum. <u>Alternatively</u>, using Second Derivative Test, $\dfrac{d^2A}{dr^2} = \dfrac{60\pi}{r^3} + \dfrac{3\pi}{2} = 3\pi + 1.5\pi = 4.5\pi > 0 \quad \therefore$ total area is a minimum.
5 (i)	$y = \dfrac{k}{1+2x} \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{-2k}{(1+2x)^2}$ When $x = 0$, $y = k$ and $\dfrac{dy}{dx} = -2k$ Equation of normal is $y - k = \dfrac{1}{2k}(x-0)$ i.e. $y - k = \dfrac{x}{2k}$ or $y = \dfrac{x}{2k} + k$ Graph of $y = \dfrac{k}{1+2x}$



	Area = $\int_0^2 \frac{x}{2k}+k-\frac{k}{1+2x}\,\mathrm{d}x$ $=\left[\frac{x^2}{4k}+kx-\frac{k}{2}\ln(1+2x)\right]_0^2$ $=\left[\frac{4}{4k}+2k-\frac{k}{2}\ln(5)\right]-0$ $=\frac{1}{k}+2k-\frac{k}{2}\ln(5)$ units²
5 (ii)	At $x = k$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2k}{(1+2k)^2}=\ell$ Then $-2k=\ell\left(1+4k+4k^2\right)$ giving $4\ell k^2+(4\ell+2)k+\ell=0$ Discriminant = $(4\ell+2)^2-4(4\ell)\ell\geq 0$ $16\ell^2+16\ell+4-16\ell^2\geq 0$ giving $\ell\geq-\frac{1}{4}$ $\therefore \ell\geq-\frac{1}{4},\ \ell\neq 0$

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6	Let X be the random variable "the duration of a pregnancy". $X \sim \mathrm{N}\left(\mu,\sigma^2\right)$ $\mathrm{P}(X<240)=0.05$ $\mathrm{P}\left(Z<\frac{240-\mu}{\sigma}\right)=0.05$ $\frac{240-\mu}{\sigma}=-1.64485$ $240-\mu=-1.64485\sigma$ --------(1) $\mathrm{P}(X>283)=0.15$ $\mathrm{P}(X\leq 283)=0.85$ $\mathrm{P}\left(Z\leq\frac{283-\mu}{\sigma}\right)=0.85$ $\frac{283-\mu}{\sigma}=1.03643$ $283-\mu=1.03643\sigma$ ---------(2) $\therefore\ \mu=266.378=266$ (3 sf) $\sigma=16.037=16.0$ (3 sf)
7 (i)	P(2 reds from large bag) = $\left(\frac{8}{24}\right)\left(\frac{7}{23}\right)=\frac{7}{69}$
7 (ii)	P(2 reds from small bag) = $\left(\frac{5}{5+n}\right)\left(\frac{4}{4+n}\right)=\frac{2}{21}$ $\Rightarrow (5)(4)(21)=2(n+5)(n+4)$ $\Rightarrow n^2+9n-190=0 \Rightarrow (n-10)(n+19)=0$ $\Rightarrow n=10$ or $n=-19$ (rejected since $n>0$)
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7 (iii)	P(large bag | 2 reds)= $\dfrac{\left(\frac{1}{3}\right)\left(\frac{7}{69}\right)}{\left(\frac{1}{3}\right)\left(\frac{7}{69}\right)+\left(\frac{2}{3}\right)\left(\frac{2}{21}\right)}=\dfrac{49}{141}$
8 (i)	The librarian can arrange all the 1440 students in a list (either in ascending order by their school exam index number or by alphabetical order of their name). Then select a random student from the whole list to be the starting point. Thereafter select every 7th student, cycling to the start of the list if the end of list is reached, until we form a sample of 200 students for the survey.
8 (ii)	The librarian can use the CCA as the strata: sports, performing arts and clubs. Calculate the sample size for each CCA group (proportional to the relative size of the CCA groups) as shown in the table below: <table><tr><td>CCA</td><td>sports</td><td>P Arts</td><td>Club</td></tr><tr><td>Sample size</td><td>62</td><td>106</td><td>32</td></tr></table>Randomly pick the required sample size from each group to obtain the sample.
8 (iii)	A stratified sample is more appropriate since it gives a more representative sample of the students who each participates in only 1 CCA.
9 (i)	P(a student plays football, but not tennis) = 1− 0.6 − 0.1 = 0.3
9 (ii)	Let x be the probability that a student plays both football and tennis Let T be the probability that a student plays tennis Let F be the probability that a student plays football $\mathrm{P}(T\|F)=0.4 \Rightarrow \dfrac{\mathrm{P}(T\cap F)}{\mathrm{P}(F)}=0.4 \Rightarrow \dfrac{x}{x+0.3}=0.4$ $\Rightarrow x=0.4x+(0.3)(0.4) \Rightarrow 0.6x=0.12$ $\therefore\ x=0.2$
9 (iii)	P(a student plays tennis only) = 0.6 − 0.2 = 0.4
9 (iv)	P(tennis | only one sport) $\dfrac{\text{P(tennis only)}}{\text{P(only one sport)}}=\dfrac{0.4}{0.4+0.3}=\dfrac{0.4}{0.7}=\dfrac{4}{7}$
10 (a) (i)	Let X be the random variable "the number of students who forget to bring his notes, out of 15 students". $X \sim \mathrm{B}(15,0.08)$ $\mathrm{P}(X\geq 1)=1-\mathrm{P}(X=0)=0.714$ (3 sf)
10 (a) (ii)	Let Y be the rv the number of students who forget to bring his notes out of n students. $Y \sim \mathrm{B}(n,0.08)$ $\mathrm{P}(Y\geq 1)\geq 0.8$ $1-\mathrm{P}(Y=0)\geq 0.8$ $\mathrm{P}(Y=0)\leq 0.2$ $n\geq 20$ $\therefore$ smallest number of students is 20. From GC, <table><tr><td>n</td><td>P(Y = 0)</td></tr><tr><td>19</td><td>0.205</td></tr><tr><td>20</td><td>0.189</td></tr><tr><td>21</td><td>0.174</td></tr></table>
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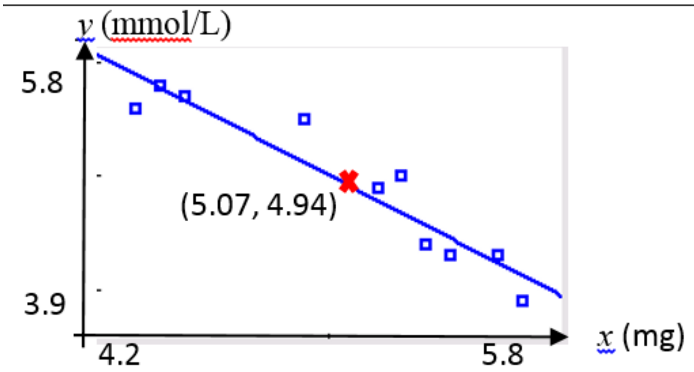
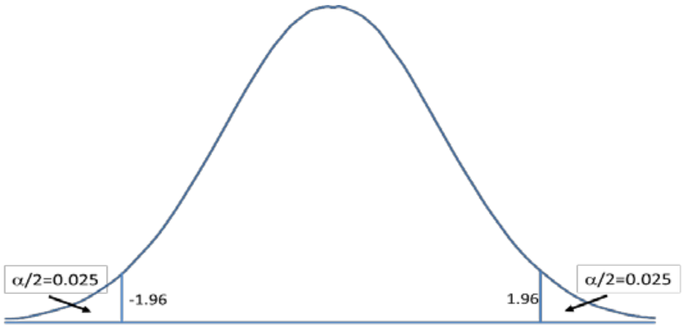
10 (b)	Let T be the random variable "the number of students who forget to bring their notes,

	out of 180 students". $T \sim \mathrm{B}(180, 0.08)$ $n = 180$ is large, $np = 180(0.08) = 14.4 > 5$ $nq = 180(0.92) = 165.6 > 5$ $\therefore$ $T \sim \mathrm{N}(14.4, 13.248)$ approximately $\mathrm{P}(T \geq 15) \xrightarrow{cc} \mathrm{P}(T > 14.5) = 0.48904 = 0.489$ (3 sf)
11 (i)	
11 (ii)	PMCC $r = -0.945$ (3 sf) Since $\|-0.945\|$ is close to 1, it means that there is a strong negative linear correlation between the amount of drug dispensed to the patient and the blood glucose level of the patient. It suggests that as more of the drug is taken, the lower the patient's blood glucose level becomes.
11 (iii)	Equation is $y = 10.541 - 1.10477x = 10.54 - 1.10x$ (3 sf)
11 (iv)	$\bar{x} = 5.07$ (3sf), $\bar{y} = 4.94$ (3 sf)
11 (v)	$y = 0.541 - 1.10477(6.5) = 3.36$ (3 sf) The estimated value of y is NOT reliable since $x = 6.5$ mg is out of the data range of x, hence we are extrapolating.
12 (i)	Let X be the rv "mass of a randomly chosen kiwi fruit". $X \sim \mathrm{N}(0.08, 0.02^2)$ $\mathrm{P}(0.07 < X < 0.09) = 0.38292 = 0.383$ (3 sf)
12 (ii)	Let Y be the rv "mass of a randomly chosen peach". $Y \sim \mathrm{N}(0.17, 0.03^2)$ Let $T = Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4 + X_5 + X_6)$ $T \sim N(3(0.17) - 6(0.08), 3(0.02^2) + 6(0.03^2))$ ie $T \sim \mathrm{N}(0.03, 0.0051)$ $\mathrm{P}(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + X_5 + X_6) = \mathrm{P}(T > 0) = 0.663$ (3 sf)
12 (iii)	Let $\mathrm{C} = 8(Y_1 + Y_2 + Y)_3 + 10(X_1 + X_2 + X_3 + X_4 + X_5 + X_6)$ $\mathrm{E}(C) = 8(3)(0.17) + 10(6)(0.08) = 8.88$ $\mathrm{Var}(C) = 8^2(3)(0.02^2) + 10^2(6)(0.03^2)$ $C \sim \mathrm{N}(8.88, 0.4128)$ $\mathrm{P}(9 < C < 10) = 0.385$ (3 sf)

13 (a) (i)	Unbiased estimate of population mean = $\bar{y}=\dfrac{3072}{600}=5.12$ Unbiased estimate of population variance $= s^2=\dfrac{1}{599}\left(16688-\dfrac{3072^2}{600}\right)=1.601602671=1.60\text{ (3 sf)}$
13 (a) (ii)	Let μ denote the population mean weight of the bags of multigrain rice. To test $H_0: \mu = 5$ Against $H_1: \mu > 5$ at 2 % sig level Under H_0, $Z=\dfrac{\bar{y}-5}{\sqrt{\dfrac{s^2}{600}}} \sim N(0,1)$ where $s^2=1.601602671$ p-value = 0.0100995741 = 0.0101 (3 sf) < 2%. Reject H_0. There is sufficient evidence at 2% level of significance to conclude that the machine dispenses more than 5kg of rice.
13 (a) (iii)	"Machine dispenses more than 5 kg of rice", i.e. reject H_0 $\therefore$ p-value < $\dfrac{\alpha}{100}$ i.e. 100 (p-value) = 1.00995741 < α The smallest level of significance to reject H_0 is 1.01 %.
13 (b) (i)	Let μ denote the population mean weight of the bags of rice. To test $H_0: \mu = 5$ Against $H_1: \mu \neq 5$ at 5% sig level Under H_0, $Z=\dfrac{\bar{x}-5}{\sqrt{\dfrac{0.153^2}{55}}} \sim N(0,1)$ by Central Limit Theorem since n is large. Do <u>not</u> reject H_0 so  $-1.9599<\dfrac{\bar{x}-5}{0.153/\sqrt{55}}<1.9599$ giving $4.96 < \bar{x} < 5.04$ (3 sf)
13 (b) (ii)	There is NO need for any assumption because Central Limit Theorem is applicable due to the large sample size.