

**ANGLO-CHINESE JUNIOR COLLEGE  
MATHEMATICS DEPARTMENT**

**MATHEMATICS  
Higher 1**

**8864 / 01**

**Paper 1**

18 August 2016

JC 2 PRELIMINARY EXAMINATION

Time allowed: **3 hours**

Additional Materials: List of Formulae (MF15)

**READ THESE INSTRUCTIONS FIRST**

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.

Write your Index number and full name on all the work you hand in.

Write in dark blue or black pen on your answer scripts.

You may use a soft pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of 7 printed pages.



**Anglo-Chinese Junior College**

**[Turn Over**

**ANGLO-CHINESE JUNIOR COLLEGE  
MATHEMATICS DEPARTMENT  
JC2 Preliminary Examination 2016**

**MATHEMATICS 8864  
Higher 1  
Paper 1**

**/ 95**

Index No:

Form Class: \_\_\_\_\_

Name: \_\_\_\_\_

Calculator model: \_\_\_\_\_

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question no.	Marks
1	/5
2	/5
3	/4
4	/8
5	/13
6	/4
7	/6

Question no.	Marks
8	/5
9	/6
10	/8
11	/9
12	/10
13	/12

Summary of Areas for Improvement			
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Presentation (P)

**Section A: Pure Mathematics [35 marks]**

1 (a) Differentiate  $7 \ln(3 - 4x)$  in terms of  $x$ , given that  $x < \frac{3}{4}$ . [2]

(b) Find the exact value of  $\int_0^2 3 - e^{2x} dx$ . [3]

2 On the same diagram, sketch the curves with equations

$$y = (0.25x + 1)(x - 2)^2 \text{ and } y = 2.5 + 2(4^{-0.5x})$$

stating clearly the equations of any asymptotes, the coordinates of any turning points and the coordinates of all points where the curves cross the coordinate axes. [3]

Hence find  $x$  such that

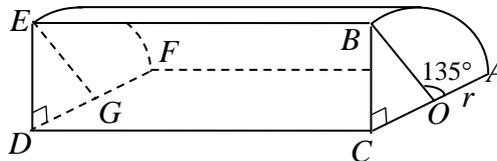
$$(0.25x + 1)(x - 2)^2 > 2.5 + 2(4^{-0.5x}). \quad [2]$$

3 Without using a calculator, solve the simultaneous equations

$$x - 2 = 10^y,$$

$$2x + 1 = 10^{2y}. \quad [4]$$

4



The diagram shows a model of the top of a building which consists of a curved roof  $ABEF$  and four flat sides  $ABC$ ,  $DEF$ ,  $BCDE$  and  $ACDF$ .  $AB$  is part of the circumference of a circle with radius  $r$  cm, centre  $O$  and angle  $AOB = 135^\circ$ .  $AOB$  and  $FGE$  are identical in shape and size.  $BCDE$  and  $ACDF$  are rectangles such that  $B$  and  $E$  are vertically above  $C$  and  $D$  respectively, and  $ACDF$  is horizontal.

Show that

(i) the length of  $BC$  is  $\frac{r\sqrt{2}}{2}$  cm, [1]

(ii) the area of  $ABC$  is  $\frac{1}{8}(2 + 3\pi)r^2$  cm<sup>2</sup>. [2]

The volume bounded by the roof and the four sides is fixed at  $5(2 + 3\pi)$  cm<sup>3</sup>. Use differentiation to find the value of  $r$  that would make the total area of the curved roof  $ABEF$  and two sides  $AOB$  and  $FGE$  a minimum value and justify why this gives a minimum value. [5]

**[Turn over**

- 5 The equation of a curve is  $y = \frac{k}{1+2x}$  where  $k$  is a constant and  $x \neq -\frac{1}{2}$ .
- (i) For  $k > 0$ , find the equation of the normal to the curve at the point where  $x = 0$ , giving your answer in terms of  $k$ . [4]
- Sketch the graph of  $y = \frac{k}{1+2x}$  and find the area bounded by the curve  $y = \frac{k}{1+2x}$ , the line  $x = 2$  and the normal to the curve at the point where  $x = 0$ , giving your answer in exact form in terms of  $k$ . [5]
- (ii) For  $k \neq 0$ , the gradient at the point on the curve where  $x = k$  is  $\ell$ . Express the relationship between  $k$  and  $\ell$  in the form of a quadratic equation. [2]
- Hence, for real values of  $k$ , find the range of possible values of  $\ell$ . [2]

### Section B: Statistics [60 marks]

- 6 The duration of pregnancy is normally distributed with mean  $\mu$  days and standard deviation  $\sigma$  days. Only 5% of all pregnancies are shorter than 240 days and 15% are longer than 283 days.
- Find the mean and variance of the distribution. [4]
- 7 (i) A large bag of sweets contains 8 red and 16 yellow sweets. Two sweets are chosen at random from the bag without replacement. Find the probability that 2 red sweets are chosen. [2]
- (ii) A small bag of sweets contains 5 red and  $n$  yellow sweets. Two sweets are chosen without replacement from the bag. If the probability that two red sweets are chosen is  $\frac{2}{21}$ , show that  $n = 10$ . [2]
- (iii) Dory has one large bag and two small bags of sweets. Assuming that each bag has an equal chance of being selected, Dory selects a bag at random and then picks two sweets without replacement. Given that two red sweets are chosen, find the probability that Dory had selected the large bag. [2]

- 8** A college has 1440 students who each participates in only 1 CCA. The breakdown of the CCA participation is as follows: 450 in sporting CCA, 760 in performing arts groups, 230 are in clubs.

A survey is to be carried out to investigate the number of hours students spend in the school library in a week. The librarian decided to sample 200 students.

- (i) Describe how he might obtain a systematic sample. [2]  
(ii) Describe how he might obtain a stratified sample, identifying the strata and finding the size of the sample taken from each of the strata. [2]  
(iii) State, with a reason, whether a systematic sample or a stratified sample would be more appropriate in this context. [1]

- 9** A group of students are asked whether they play tennis or football. The probability that a student plays tennis is 0.6. The probability that a student plays neither tennis nor football is 0.1.

- (i) Find the probability that a student plays football but not tennis. [1]

Given that a student plays football, the probability that he also plays tennis is 0.4.

- (ii) Show that the probability that a student plays both tennis and football is 0.2. [2]  
(iii) Find the probability that a student plays only tennis. [1]  
(iv) Given that a student plays only one sport, find the probability that he plays tennis. [2]

- 10** The probability that a student forgets to bring his notes is 0.08, independently of other students.

- (a) The whole class has to do a quiz if at least one of the students forgets to bring his notes.

- (i) For a class with 15 students, find the probability that the class will have to do a quiz. [2]

- (ii) For a class with  $n$  students, find the smallest number of students in the class such that the probability that the class will have to do a quiz is at least 0.8. [2]

- (b) The whole lecture group has to do a quiz if at least 15 students forget to bring notes. Using a suitable approximation, estimate the probability that a lecture group of 180 students will have to do a quiz. State the parameter(s) of the distribution that you use. [4]

**[Turn over**

- 11** Blood sugar level is the amount of glucose present in your blood at any given time. One form of measurement for blood glucose level is millimole per litre (mmol/L).

A clinical company created a new drug that they claim helps to regulate the blood sugar level for diabetics. Some diabetics were invited to try out this new drug, and 10 patients were randomly selected for a blood test. The table shows the amount of new drug,  $x$  mg, dispensed to each of the 10 patients each day and the measurement of their blood glucose level,  $y$ , mmol/L.

$x$ (mg)	5.8	5.5	5.4	5.7	5.2	5.3	4.9	4.4	4.3	4.2
$y$ mmol/L)	3.9	4.3	4.4	4.3	4.9	5.0	5.5	5.7	5.8	5.6

- (i) Draw a scatter diagram for these values, labelling the axes. [1]
- (ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]
- (iii) Find the equation of the regression line of  $y$  on  $x$ , in the form of  $y = mx + c$ , giving the values of  $m$  and  $c$  to 2 decimal places. Sketch this line on your scatter diagram. [2]
- (iv) Find  $\bar{x}$  and  $\bar{y}$ , and mark the point  $(\bar{x}, \bar{y})$  on your scatter diagram. [2]
- (v) Use the equation of your regression line to calculate an estimate of the blood glucose level if the new drug dispensed is 6.5 mg. Comment on the reliability of your estimate. [2]
- 12** The masses, in kilograms, of kiwi fruit and peaches sold in a supermarket have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Kiwi fruit	0.08	0.02
Peach	0.17	0.03

- (i) Find the probability the mass of a kiwi fruit chosen at random is within  $\pm 0.01$  kg of the mean mass of kiwi fruit. [2]
- (ii) Three peaches and six kiwi fruits are chosen at random. Find the probability that the total mass of the three peaches is greater than the total mass of the six kiwi fruits. [4]
- (iii) Peaches cost \$8 per kilogram and kiwi fruits cost \$10 per kilogram. Find the mean and the variance of the total cost of three peaches and six kiwi fruits. Hence find the probability that the total cost is between \$9 and \$10. [4]

- 13 (a)** A company distributes rice in packets. The mass of a packet of rice follows a normal distribution. The mass of each packet of rice is denoted by  $Y$  kg. The masses of a random sample of 600 packets of rice are summarised by

$$\sum y = 3072 \text{ and } \sum y^2 = 16688.$$

- (i) Calculate the unbiased estimates of the mean and variance of  $Y$ . [2]

The mean mass of rice in a packet is 5 kg. The managers wants to check if their machinery has been dispensing more than 5 kg of rice.

- (ii) Carry out this hypothesis testing at the 2% significance level. [4]

- (iii) Find the least level of significance at which this sample would indicate that the machinery has been dispensing more than 5 kg of rice. [1]

- (b)** Another company also wanted to assess if their machinery has been accurate in dispensing rice into 5 kg per packet. For this company, the mean mass of a packet of rice is 5 kg and the standard deviation 0.153 kg. The manager takes a random sample of 55 packets and carries out a test, at the 5% significance level, of whether the machinery dispenses rice at the stated 5 kg per packet.

- (i) The sample mean mass of a packet of rice is denoted by  $\bar{x}$ . Use an algebraic method to calculate the set of values of  $\bar{x}$  for which the null hypothesis would not be rejected. (Answers obtained by trial and improvement from a calculator will obtain no marks). [4]

- (ii) State, giving a reason, whether any assumption about the population is needed in order for the test to be valid. [1]

**~ End of Paper ~**