ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT

MATHEMATICS
Higher 2

9740 / 02

Paper 2

22 August 2016

JC 2 PRELIMINARY EXAMINATION

Time allowed**: 3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.
Write your Index number and full name on all the work you hand in.
Write in dark blue or black pen on your answer scripts.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

Anglo-Chinese Junior College

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ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2016

MATHEMATICS 9740
Higher 2
Paper 2

/ 100

Index No: ☐ ☐ ☐ ☐   Form Class: _____

Name: _____

Calculator model: _____

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/3
2	/14
3	/8
4	/15
5	/3
6	/5
7	/10
8	/8
9	/15
10	/9
11	/10

Summary of Areas for Improvement			
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Qn wrongly (R)**	**Presentation (P)**

Section A: Pure Mathematics [40 marks]

1 A triangular region R is drawn on a large sheet of graphing paper marked in 1mm squares. The region R is bounded by the x-axis, the line $y = \frac{1}{20}x$ and the line $y = -\frac{1}{20}x + 40$. The scales on both the axes are such that 1 mm represents 1 unit.

By using the table below or otherwise, find the number of complete 1mm squares which lie inside region R. [3]

Range of x	Number of complete 1 mm squares which lies inside region R
$20 \leq x \leq 40$	
$40 \leq x \leq 60$	
$60 \leq x \leq 80$	
$\vdots$	

2 The parametric equations of curve C are

$$x = a\cos^3 t, \quad y = a\sin^3 t \quad \text{for } 0 \leq t \leq \pi,$$

where a is a positive constant.

(i) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis and the coordinates of the points at which the tangent is parallel to the y-axis. [5]

(ii) Hence sketch the curve C. [1]

(iii) If the cartesian equation of curve C is $y = f(x)$, use the curve in part **(ii)** to sketch the graphs of

        (a) $y = \dfrac{1}{f(x)}$,   **(b)** $y = f'(x)$,

stating in each case, the equations of any asymptotes and the coordinates of any points of intersection with the axes. [4]

(iv) The point P on the curve has parameter p. Show that the equation of tangent at P is

$$x\sin p + y\cos p = a\sin p \cos p.$$ [2]

(v) The tangent at P is perpendicular to the tangent at another point Q, on the curve.

If $p = \dfrac{\pi}{3}$, find the value of the parameter t at point Q. [2]

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3   The functions h and g are defined by

$$h : x \mapsto e^{|2x+1|} + 1, \ x \in \mathbb{R}, \ x \le k,$$

$$g : x \mapsto \begin{cases} 2x & \text{for } 0 \le x \le \frac{1}{2}, \\ 2-2x & \text{for } \frac{1}{2} \le x \le 1. \end{cases}$$

(i)   Given that the function h has an inverse, state the greatest value of k. Find $h^{-1}(x)$ and write down the domain of h^{-1}.   [4]

(ii)   Explain why the composite function gg exist.   [1]

(iii)   Sketch the graph of $y = gg(x)$.   [2]

(iv)   Given that $f : x \mapsto e^{|2x+1|} + 1, \ x \in \mathbb{R}$, find the range of fg exactly.   [1]

4   The equations of three planes p_1, p_2, p_3 are

$$2x + 3y - 6z = 10,$$
$$-2x - 3y + 6z = a,$$
$$x + y + bz = 5,$$

respectively, where a, b are constants.

The planes p_1 and p_3 intersect in the line l with cartesian equation $\dfrac{5-x}{3} = \dfrac{y}{4} = z$.

(i)   Show that $b = -1$.   [2]

(ii)   The point S lies on p_1 and the point R has coordinates $(-2, \ 4, \ 1)$. Given that RS is perpendicular to p_3, find the coordinates of S.   [4]

The planes p_1 and p_2 are $\dfrac{8}{7}$ units apart.

(iii)   Given that $a < 0$, find the possible values of a.   [4]

(iv)   The point P with coordinates $(5, \ 2, \ c)$ lies on p_1. Find the value of c.   [1]

(v)   The point F is the foot of the perpendicular from P to the line l. The point Q is the reflection of F in the plane p_2. Find the distance PF and hence find the area of triangle FPQ.   [4]

Section B: Statistics [60 marks]

5 Florida fitness club wants to carry out a survey to find out from their members the facilities that the club can improve on. The club has a list of all the 15000 members' names.

    (i) Describe how to obtain a systematic sample of 500 members from the list to take part in the survey. [2]

    (ii) State one disadvantage of using a systematic sample in this context. [1]

6 A factory manufactures rectangular glass panels. The length and breadth of each panel, in cm, are modelled as having independent normal distributions with means and standard deviations as shown in the table.

Glass Panel	Mean (cm)	Standard Deviation (cm)
Length	300	0.5
Breadth	150	0.2

The probability that the total perimeter of 2 randomly selected glass panels exceeds the mean length of n randomly selected glass panels by more than 1501cm is less than 0.2576. Find the least value of n. [5]

7 The mean number of guests checking into a hotel in an hour is 3.6 and can be modelled by a Poisson distribution.

    (i) Find the probability that not more than 4 guests checked into the hotel in a given hour. [1]

    (ii) Given that three non-overlapping one-hour blocks are chosen at random, find the probability that one of the blocks has not more than 4 guests checking into the hotel and the remaining two blocks have no guests checking into the hotel. [2]

    (iii) Given that each day consists of 24 non-overlapping one-hour blocks. Use a suitable approximation, to find the probability that between 85 and 90 guests checked into the hotel in a particular day. State the parameters of the distribution that you use. [3]

    (iv) The probability of at least n one-hour blocks in a day of 24 non-overlapping one-hour blocks has not more than 4 guests checking into the hotel is less than 0.124. Find the least value of n. [3]

    (v) Explain why the Poisson distribution may not be a good model for the number of guests checking into the hotel in a year. [1]

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8 Tandao Café has an outlet at North Vista and another outlet at South Parc. On weekdays, the waiting time during lunch periods in each outlet follows a normal distribution with mean μ minutes.

(i) Hono has lunch regularly at the North Vista outlet. On 10 randomly selected weekdays, his waiting times per visit were recorded, in minutes, as follows:

$$49, \ 38, \ 43, \ 70, \ 45, \ 51, \ 57, \ 85, \ 39, \ 44$$

Test, at the 10% significance level, whether the mean waiting time is less than one hour. [4]

(ii) Lulu has lunch regularly at the South Parc outlet. On 56 randomly selected weekdays, her waiting times per visit were recorded, in minutes. It was found that the sample mean waiting time is $\bar{t}$ minutes and the sample variance is 69.8 minutes^2. A test is to be carried out at the 5% level of significance to determine whether the average waiting time at the South Parc outlet is not one hour.

Find the range of values of $\bar{t}$ for which the result of the test would be that the null hypothesis is rejected, leaving your answers in 2 decimal places. (Answers obtained by trial and improvement from a calculator will obtain no marks.) [4]

9
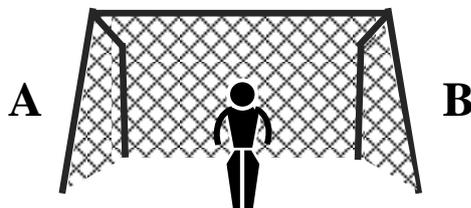
(a) The diagram above shows a goalkeeper and a front view of a goal post labelled A on one side and B on the other side. Aaron is a goalkeeper in a football club. Based on his past experiences as a goalkeeper in penalty shoot-outs, the probability that he dives to side B is 0.72. In a particular match, Aaron's team went into a penalty shoot-out. The probability that a penalty kicker kicks the ball to side B is p, where $0 < p < 1$. Assume that Aaron's choice of direction to dive is independent of the penalty kicker's choice of direction to kick the ball.

(i) Show that the probability Aaron dives in the same direction as the ball is kicked is $0.44p + 0.28$. [1]

(ii) If Aaron dives in the same direction as the ball is kicked, the probability that he saves the ball is 0.4. Find, in terms of p, the probability that Aaron fails to save the ball. [3]

Hence find the values between which this probability must lie. [1]

(b) In any football match, the expected number of saves made by Aaron is 4.8, with a standard deviation of 1.2. Find the probability that in 50 matches, the average number of saves per match made by Aaron is less than 5. [2]

(c) Aaron can either be a goalkeeper or a forward in a match, depending on the strategy that the team uses. In the team, there is another goalkeeper, 6 defenders, 5 midfielders and 5 forwards. Altogether, there are 18 members in the team.

    (i) Find the number of ways to select 4 defenders, 4 midfielders, 2 forwards and a goalkeeper from the team, given that Aaron is selected. [3]

After a match, the team stands in a straight line to take a photo.

    (ii) Find the number of ways such that all the defenders are standing alternately with all the midfielders. [2]

The team is then asked to sit in a circle for a debrief.

    (iii) Find the probability that the two goalkeepers sit opposite each other given that a group consisting of 2 particular midfielders and 6 defenders are seated together. [3]

10 (i) Sketch a scatter diagram that might be expected when h and s are related approximately as given in each of the models (A) and (B) below. In each model, your diagram should include 6 points, approximately equally spaced with respect to h, and all h- and s-values positive. The letters a, b, c and d represent constants.

    (A) $s = a + b \ln h$, where a is negative and b is positive.

    (B) $s = c + \dfrac{d}{h}$, where c is positive and d is negative. [2]

A company recently launched a new product in Singapore and wanted to know more about the relationship between the number of promoters, h, and the product's monthly sales, s, in Singapore dollars. They collected data for the past 9 months and the results are given in the table.

h	50	60	70	80	90	100	110	120	130
s	40 000	47 000	52 000	55 000	57 800	60 000	61 500	62 500	63 000

(ii) Draw a scatter diagram for these values, labelling the axes. [1]

(iii) Comment on whether a linear model would be appropriate, referring to both the scatter diagram and the context of the question. [2]

[Turn over

(iv) It is required to estimate the number of promoters needed to achieve a monthly sales of $75,000. Using an appropriate model in part **(i)** to find the equation of the suitable regression line, correct to 3 decimal places. Use your equation to find the required estimate, correct to the nearest integer. [2]

(v) Comment on the reliability of your estimate. [1]

(vi) Given that 1 US dollar = 1.34 Singapore dollars, re-write your equation from part **(iv)**, correct to 3 decimal places, so that it can be used to estimate the number of promotors when the product's monthly sales is given in US dollars. [1]

11 The manager of a car show room wants to study the number of cars sold by the 2 car salesman under his charge. The number of potential car-buyers that they meet in a particular week and the average probabilities that each salesman is successful in closing a deal with each customer is given in the table below.

Salesman	Number of potential car-buyers	Probability of closing a deal
X	60	0.2
Y	50	0.3

(i) It is assumed that the deals closed are independent of one another. State, in context, another assumption needed for the number of deals closed by a car salesman to be well modelled by a binomial distribution. [1]

(ii) Explain why the assumption that the deals closed are independent of one another may not hold in this context. [1]

Assume now that the assumptions stated in part **(i)** do in fact hold and the deals closed by salesman X is independent of the deals closed by salesman Y.

(iii) Use suitable approximations to find the probability that both salesmen collectively closed a total of more than 20 deals in a particular week. State the parameters of the distributions that you use. [4]

A new salesman joined the company. During his probation week, he met 60 potential car-buyers. The number of car deals he closed during his probation week is denoted by C with the distribution $B(60, p)$.

(iv) Given that $P(C = 30) = 0.03014$. Find an equation for p. Hence find the value of p, correct to 1 decimal place, given that $p < 0.5$. [2]

(v) Given that $p = 0.05$, use a suitable approximation, which should be stated, to find the probability that he sold more than 4 cars. [2]

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