ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT

MATHEMATICS
Higher 2

9740 / 01

Paper 1

18 August 2016

JC 2 PRELIMINARY EXAMINATION

Time allowed**: 3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.
Write your Index number and full name on all the work you hand in.
Write in dark blue or black pen on your answer scripts.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages.

Anglo-Chinese Junior College

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ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2016

MATHEMATICS 9740
Higher 2
Paper 1

/ 100

Index No: ☐☐☐☐ Form Class: ___________

Name: ___________________________

Calculator model: ___________________

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/5
2	/5
3	/5
4	/9
5	/4
6	/8
7	/8
8	/6
9	/9
10	/11
11	/7
12	/12
13	/11

Summary of Areas for Improvement			
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Presentation (P)

1 Without the use of a calculator, solve the inequality $\dfrac{2w-5}{w^2-3}>0$. [2]

Hence solve $\dfrac{\left(2|y|-5\right)\sin x}{y^2-3}\le 0$, given that $\pi < x \le \dfrac{3\pi}{2}$. [3]

2 The equation of the curve C is given by $y=\ln x$. The line ℓ with the equation $y=\dfrac{x}{\mathrm{e}}$ is tangential to the curve C at the point $\left(\mathrm{e},1\right)$. The region R is bounded by the curve C, the line ℓ and the x-axis. The solid S is formed by rotating the region R through 2π radians about the x-axis. Find the exact volume of the solid S in terms of π and e. [5]

3 **(i)** Every year Warren Gate's net worth increases by 100% of the previous year. His net worth was estimated to be \$1 000 000 on 31st December 1993. In what year will his fortune first surpass 2.5 billion dollars (1 billion = 10^9). [3]

(ii) On 1st January 2005, Warren Gates deposits \$100 000 in an investment account and receives an interest of \$1000 on 31st December 2005. After that the amount of interest earned at the end of the year is 1.5 times the amount of interest earned in the previous year. Taking year 2005 as the first year, find the amount of savings that Warren Gates has in his account at the end of the 15th year giving your answer to the nearest integer. [2]

4 **(a)** It is given that $\mathrm{g}(x)=\dfrac{1}{\cos(\frac{\pi}{4}+x)\cos(\frac{\pi}{4}-x)}$ where x is sufficiently small for x^3 and higher powers of x to be neglected.

Show that $\mathrm{g}(x)\approx 2+ax+bx^2$, where a and b are constants. [3]

Comment on the value of m for this expression $\int_{-m}^{m}\mathrm{g}(x)\,\mathrm{d}x \approx \int_{-m}^{m}(2+ax+bx^2)\,\mathrm{d}x$ to be valid. [1]

(b) Find the first four non-zero terms of the expansion of $\left(1-x^2\right)^{-\frac{1}{2}}$ in ascending powers of x where $|x|<1$. [2]

Hence find the first four non-zero terms of the Maclaurin's series for $\cos^{-1}x$ in ascending powers of x. [3]

[Turn Over

5 A sequence $u_1,\ u_2,\ u_3,\ \ldots$ is given by

$$u_1 = 1 \text{ and } \frac{1}{3^r}\left(\frac{u_{r+1}}{3} - u_r\right) = 2r \text{ for all } r \geq 1.$$

Use the method of differences to prove that $u_{n+1} = 3^n(3n^2 + 3n + 1)$ for all $n \geq 1$. [4]

6 An investor deposits \$$K$ in a bank account. The bank offers an annual interest rate of 5% compounded continuously. No further deposits are made. The amount of money in the account at time t years is denoted by M. Both M and t are taken to be continuous variables. Money is withdrawn at a continuous rate of \$4000 per year. Set up a differential equation and show that for $t > 0$, $\frac{\mathrm{d}M}{\mathrm{d}t} = aM + b$, where a and b are constants to be determined. [1]

For $t > 0$, find M in terms of t and K. [4]

On a single clearly labelled diagram, show the graph of M against t for

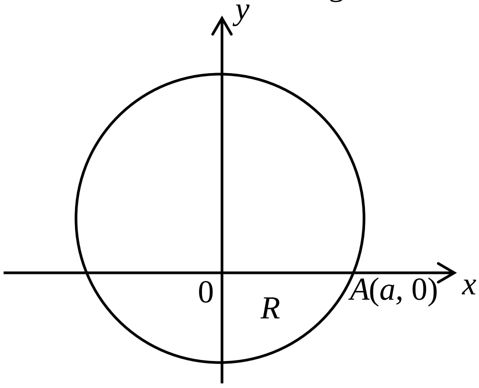
(i) $K > 80\ 000$, [1]

(ii) $K < 80\ 000$. [1]

Hence state the condition for which the money deposited initially will be completely withdrawn in a finite period of time. [1]

7 **(i)** Use the substitution $x = 5\sin\theta$ to find $\int \sqrt{25 - x^2}\,\mathrm{d}x$. [5]

(ii) The circle with equation $x^2 + (y - b)^2 = 25$ where $0 < b < 5$, cuts the positive x-axis at $A(a,\ 0)$. The region R is bounded by the x and y axes, and the part of the circle lying in the fourth quadrant as shown in the diagram below.



Use your result in **(i)** to find the area of the region R in terms of a. [3]

8 The complex number z is given by $z = k + \mathrm{i}$ where k is a non-zero real number.

(i) Find the possible values of k if $z = k + \mathrm{i}$ satisfies the equation $z^3 - \mathrm{i}z^2 - 2z - 4\mathrm{i} = 0$. [3]

(ii) For the complex number z found in part **(i)** for which $k > 0$, find the smallest integer value of n such that $|z^n| > 100$ and z^n is real. [3]

9 Use the method of mathematical induction to prove that

$$\sum_{r=1}^{n} \frac{3r+1}{r(r+1)(r+2)} = \frac{n(7n+9)}{4(n+1)(n+2)}, \text{ for all } n \geq 1. \quad [5]$$

(i) Show that $\dfrac{n(7n+9)}{4(n+1)(n+2)} < \dfrac{7}{4}$. [2]

(ii) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{3r}{(r+1)^3} < \frac{7}{4}$. [2]

10 The curve C has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x^2 - 4k^2}{x - k}$. It is given that k is a constant and $x \neq k$.

Find the set of possible values that y can take. [3]

For the case $k > 1$,

(i) Sketch the graph of C, stating in terms of k, the coordinates of any points of intersection with the axes and equations of any asymptotes. [3]

(ii) Hence find $\displaystyle\int_{-1}^{1} \mathrm{f}(|x|)\,\mathrm{d}x$ in terms of k. [3]

(iii) The graph of curve C is transformed by a scaling of factor 2 parallel to the x-axis, followed by a translation of $2k$ units in the negative x-direction. Find the equation of the new curve. You need not simplify your answer. [2]

11 Referred to the origin O, the position vectors of the points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. Given that $\mathbf{a} \times \mathbf{b} = 4\mathbf{a} \times \mathbf{c}$, where $\mathbf{b} \neq 4\mathbf{c}$ and $\mathbf{a}$ is a non-zero vector,

(i) show that $\mathbf{b} - 4\mathbf{c} = \alpha\mathbf{a}$ where α is a scalar. [1]

(ii) Hence evaluate $|\mathbf{b} \times \mathbf{c}|$, given that the area of triangle OAB is $\sqrt{126}$ and $\alpha = \sqrt{3}$. [2]

(iii) Give the geometrical meaning of $|\mathbf{b} \times \mathbf{c}|$. [1]

It is also given that $\mathbf{b}$ is a unit vector, $|\mathbf{a}| = 5$, $|\mathbf{c}| = 2$ and $\mathbf{b} - 4\mathbf{c} = \sqrt{3}\mathbf{a}$.

(iv) By considering $(\mathbf{b} - 4\mathbf{c}) \cdot (\mathbf{b} - 4\mathbf{c})$, find the angle between $\mathbf{b}$ and $\mathbf{c}$. [3]

[Turn Over

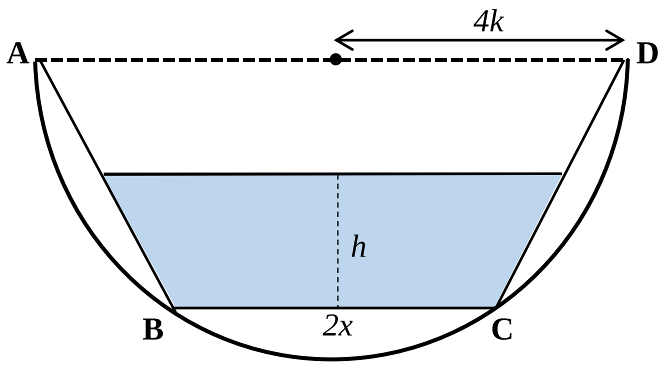
12 **(i)** Solve the equation $z^5 - \mathrm{i} = 0$, giving the roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. Show the roots z_1, z_2, z_3, z_4 and z_5 on an Argand diagram where $-\pi < \arg(z_1) < \arg(z_2) < \arg(z_3) < \arg(z_4) < \arg(z_5) \le \pi$. [5]

(ii) Find the exact cartesian equation of the locus of all points z such that $|z - z_2| = |z - z_3|$ and sketch this locus on an Argand diagram. Find the least possible value of $|z - z_1|$. [4]

(iii) Sketch on the same Argand diagram in **(ii)**, the locus $\arg(z - z_1) = \arg(z_4)$. [1]

(iv) Find the complex number z that satisfy the 2 equations $|z - z_2| = |z - z_3|$ and $\arg(z - z_1) = \arg(z_4)$, giving your answer in the form $a + \mathrm{i}b$. [2]

13



The diagram shows the cross-section of a container. It is in the shape of a semicircle of fixed radius $4k$ metres with a hole in the shape of a trapezium *ABCD*.

(i) If $BC = 2x$ metres, show that the area S of the trapezium *ABCD* is given by $S = (x+4k)\sqrt{16k^2 - x^2}$. [2]

(ii) Use differentiation to show that the area of the trapezium is maximum when $x = 2k$ metres. [4]

It is given that $x = 2k$ metres and the length of the container is given as 3 metres. This container is filled with water at a constant rate of 0.2 m^3/s. At time t seconds the depth of water in the container is h metres as shown.

(iii) Show that the volume V of water in the container is given by $V = 3h\left(4k + \dfrac{h}{\sqrt{3}}\right)$. [2]

(iv) Find, in terms of k, the rate at which the depth is increasing at the instant when the depth is $k\sqrt{3}$ metres. [3]

- End of Paper -